Samuel Neaman Institute
FOR ADVANCED STUDIES IN SCIENCE AND TECHNOLOGY

Technion
Israel Institute of Technology

Investment Policies in Defense R&D Programs

Oren Setter • Asher Tishler

6 WORKING PAPER SERIES - ECONOMICS OF NATIONAL SECURITY

The Samuel Neaman Institute

for Advanced Studies in Science and Technology

Technion, Israel Institute of Technology, Technion City, Haifa Israel 32000

The Samuel Neaman Institute for Advanced Studies in Science and Technology is an independent multi-disciplinary public-policy research institute, focused on issues in science and technology, education, economy and industry, and social development. As an interdisciplinary think-tank, the Institute draws on the faculty and staff of the Technion, on scientists from other institutions in Israel, and on specialists abroad. The Institute serves as a bridge between academia and decision makers in government, public institutions and industry, through research, workshops and publications.

The Samuel Neaman Institute activities are at the interface between science, technology, economy and society. Therefore, the natural location for the Institute is at the Technion, which is the leading technological university in Israel, covering all the areas of science and engineering. This multi-disciplinary research activity is more important today than ever before, since science and technology are the driving forces for growth and economic prosperity, and they have a significant influence on the quality of life and a variety of social aspects.

The Institute pursues a policy of inquiry and analysis designed to identify significant public policy issues, to determine possible courses of action to deal with the issues, and to evaluate the consequences of the identified courses of action.

As an independent not-for-profit research organization, the Institute does not advocate any specific policy or embrace any particular social philosophy. As befits a democratic society, the choices among policy alternatives are the prerogative and responsibility of the elected representatives of the citizenry. The Samuel Neaman Institute mission is to contribute to a climate of informed choice. Each research program undertaken by the Institute is expected to be a significant scholarly study worthy of publication and public attention. All the research done by the institute, as well as the many workshops and other publications are disseminated free of charge on the website of the institute: http://www.neaman.org.il/

Origins

The Institute was established by the initiative of Mr. Samuel Neaman, a prominent U.S. businessman noted for his insightful managerial concepts and innovative thinking, as well as for his success in bringing struggling enterprises to positions of fiscal and marketing strength. He devoted his time to the activities of the Institute, until he passed away in 2002.

Organization

The Director of the Institute, appointed jointly by the President of the Technion and by the Chairman of the Institute Board, is responsible for formulating and coordinating policies, recommending projects and appointing staff. The current Director is Prof. Nadav Liron and the Board of Directors is chaired by Prof. Zehev Tadmor. The Board is responsible for general supervision of the Institute, including overall policy, approval of research programs and overseeing financial affairs. An Advisory Council made up of members of the Technion Senate and distinguished public representatives, reviews research proposals and consults on program development.

Investment Policies in Defense R&D Programs[1]

Oren Setter and Asher Tishler, Tel Aviv University

Abstract

Investment in advanced defense technologies is a prominent characteristic of modern armed forces. The paper examines the optimal investment policy in developing such technologies, accounting for their S-shaped progress profile and the stochastic nature of the R&D process. We show that the optimal investment is a discontinuous function of the available budget, and that its dependence on technological uncertainty is non-monotonous. We further show that maintaining the flexibility to adjust investments along the R&D program is beneficial.

[1] We are grateful to Sarit Markovich and to seminar participants in Tel Aviv University and the University of Iowa for their helpful comments. This research was supported by the Economics of National Security Program, Samuel Neaman Institute.

1. Introduction

Technology is at the heart of modern armed forces, as exemplified by technologies such as stealth, precision guidance, satellite navigation and command and control systems. Accordingly, significant shares of national defense budgets are allocated to research and development (R&D) activities aimed at developing new and improved systems. For example, the USA allocated about 16% of its $420 billion defense budget in 2005 (DoD, 2004) to defense R&D activities, and an additional 18% were devoted to the procurement of systems which are the outcome of R&D activities.

This paper focuses on investment policies in advanced technologies, the term "advanced" referring to technologies resulting from highly-innovative long-term R&D efforts (as opposed to small incremental improvements). Investment policies in advanced defense technology differ among countries. While some countries hardly invest in advanced R&D, but rather focus on incremental improvements (or on imports alone), others invest in highly uncertain R&D programs. A country may even have different policies for different defense programs. In some cases it may opt to follow in the footsteps of other countries, and develop similar defense systems, while in other cases it may prefer to develop state-of-the-art technologies and, thus, be among the world leaders in those technologies.

The objective of this paper is to analyze the problem of investment in advanced R&D programs within a defense acquisition context. To that end, we develop an analytical model which captures the defining characteristics of such programs, such as an S-shaped value function, technological uncertainty and a lengthy R&D process.

Using this model we solve for the optimal investment policy, and characterize its prescriptions under different conditions.

The results of the paper contribute to the literature in several ways. First, we prove the existence of a budget threshold for investment in advanced technologies: below the threshold, zero investment is optimal; above the threshold, however, a sizable investment is optimal. Second, we find that investment in highly uncertain defense R&D is expected in both small and large countries, while medium-sized countries are expected to invest in mature technologies. Finally, we demonstrate that the lengthy nature of defense R&D processes allows an "aggressive" investment policy in the early stages of the R&D program, balanced by the option to adjust investments later on after the appearance of early R&D results.

The paper proceeds as follows. We provide a short literature review in Section 2, and present a basic model of investment in advanced technologies in a deterministic setting in Section 3. Section 4 extends the model to a stochastic setting, incorporating technological uncertainty, dynamic considerations being introduced in Section 5. Section 6 concludes and provides directions for future research.

2. Literature Review

The value of technology is often characterized as having an S shape (Christensen, 1992). At first, one develops the technological infrastructure, yielding only small operational benefits. Afterwards, benefits grow fast, and the technology enjoys increasing returns. Finally, as the technology matures its marginal benefit declines. For example, Setter and Tishler (2004) show that the value of the integrative technologies used in the US military for command and control applications exhibits an S shape, having crossed the inflection point only recently. Loch and Kavadias

(2002) examine optimal investment policies when technologies have either increasing returns or decreasing returns. They show that in the former case the budget should be allocated to a single technology, while in the latter it should be allocated between various technologies according to their total marginal returns. Setter and Tishler (2004) extend the model by examining the optimal investment policy along the entire life cycle of the technology, but use a specific functional form to model the S curve. The current paper further generalizes the model of Setter and Tishler (2004) by allowing for a general S-shaped function.

There is general agreement that the rate of progress of advanced technologies is highly uncertain (Dasgupta and Maskin, 1987; Pennings and Lint, 1997). By definition, R&D is an effort that aims to achieve some goal, or level of performance, for the first time. On the basis of past experience, it is, therefore, very difficult, and sometimes impossible, to accurately predict the outcome of some given R&D effort. This is especially true for advanced, highly innovative, technologies, which themselves differ in their level of uncertainty, depending on several factors: feasibility and existence of the technology, local experience with the technology and the expected technological progress (that is, an incremental improvement or a technological breakthrough).

For a given R&D effort, the level of technological progress obtained can range from total failure to shining success, and can take any value in between. Furthermore, while the outcome of the R&D effort is more likely to end up near the target level, it may also be far below or above it. This view of technological uncertainty differs from many models of R&D, in which R&D efforts may either "fail" or "succeed" (Dixit, 1987): failure indicating lack of any progress, and success implying that the target level has been obtained. While the latter modeling approach may hold for scientific discoveries, the outcome (success level) of most R&D programs is better characterized by a continuous random variable (Grossman and Shapiro, 1986).

3. Basic Model

Consider a defense decision maker who allocates a budget B to the acquisition (R&D and procurement) of a weapon system of quality q and quantity x, in order to maximize its value, denoted V. We assume quantity and quality to fully describe the characteristics of this system and its value. Formally, we define the value of a weapon system as:

$$V(x,q) = f(x)\cdot(\varphi + g(q)) \tag{1}$$

where $f(\cdot)$ is the benefit function of quantity. We assume that $f(0)=0$ and that $f(\cdot)$ is positive, increasing and concave for every $x>0$. $\varphi > 0$ is a baseline quality level, which does not require any further research and development. Without loss of generality we normalize it to 1 in the remainder of the paper. $g(\cdot)$ is the benefit function of quality, assumed positive, increasing, S shaped (convex/concave) and converges asymptotically to g_{max} as q approaches infinity. Formally, $g(q) > 0$, $g_q > 0$, $g_{qq} \geq 0$ for $q \in [0, \tilde{q}]$ and $g_{qq} < 0$ for $q > \tilde{q}$.

The model assumes quantity to have a linear cost function and quality to have a convex cost function[2]. For simplicity, we take the unit price of quantity to be 1, and denote the cost function of quality by $c(q)$; hence the budget constraint takes the following form:

[2] Setter (2004) provides three reasons for the convexity of R&D costs: the sequential nature of R&D, the trial-and-error nature of R&D and the inelastic supply of R&D personnel (Goolsbee, 1998).

$$x + c(q) \leq B, \tag{2}$$

where $c(q)$ is positive, increasing and convex.

The decision maker thus has to maximize (1), by choosing x and q, subject to the budget constraint (2) and to non-negativity constraints. The budget constraint is binding; thus the optimization problem may be rewritten as:

$$\max_q \tilde{V}(q) = \tilde{f}(q) \cdot (1 + g(q)), \tag{3}$$

where $q \in \left[0, c^{-1}(B)\right]$, and $\tilde{f}(q) \equiv f\left(B - c(q)\right)$. It is straightforward to verify that $\tilde{f}(\cdot)$ is positive, *decreasing* and concave.

If an internal solution, denoted $\hat{q}$, to Program (3) exists, it satisfies the following first-order condition, derived by differentiating the logarithm of the objective function:

$$\frac{g_q}{1+g} = \frac{\tilde{f}_q}{\tilde{f}} \tag{4}$$

The LHS of Equation (4) is the marginal benefit of quality (in operational terms), whereas the RHS of Equation (4) is the marginal cost of quality (in operational terms), resulting from the implied decrease in quantity. At the optimum, the marginal benefit (denoted $MB(q)$) equals the marginal cost (denoted $MC(q)$). The existence and uniqueness of the internal solution are, however, not trivial, as the objective function is not globally concave. Proposition 1 provides necessary and sufficient conditions for the existence of a (unique) internal solution.

Proposition 1: An internal optimal solution to (3) exists if, and only if:

$$MB\left(q^*\right) > MC\left(q^*\right) \tag{5}$$

where q^* is the solution of

$$\frac{\partial MB(q)}{\partial q} = \frac{\partial MC(q)}{\partial q}. \tag{6}$$

If the solution exists, it is unique. The globally optimal solution is the greater between $\tilde{V}(\hat{q})$ and $\tilde{V}(0)$.

Proof: All proofs appear in the Appendix.

Intuitively, $MB\ (q) < MC\ (q)$ for large enough values of q; thus if we show the opposite to hold for some smaller value of q, then there is also a value of q for which they are equal. q^* is, by definition, the point where the difference between $MB(q)$ and $MC\ (q)$ is maximal. Hence, if condition (5) holds, an internal solution exists. If, however, it does not hold, then by definition there is no such internal solution.

An important implication of Proposition 1 is the existence of a budget threshold for investment in *advanced*[3] R&D.

Proposition 2: If the regularity condition

$$\frac{\partial MB(0)}{\partial q} > \frac{\partial MC(0)}{\partial q} \tag{7}$$

holds, there exists a budget threshold, B^*, such that:

$$B \le B^* \Rightarrow \hat{q} = 0 \text{ and } B > B^* \Rightarrow \hat{q} > q^* > 0 .$$

[3] That is, if $g(q)$ is convex "enough".

If the budget is smaller than the threshold, a zero investment in R&D is optimal. Above the threshold, however, a *sizable* investment is optimal. Hence, investment in advanced R&D is not a continuous function of the available budget. Thus, one would expect to see countries that do not spend on advanced R&D at all, while the expenditures of others on it are considerable. *Small* expenditures on advanced R&D should be rare. Intuitively, this result stems from the S shape of the technology improvement function. Small investments in advanced R&D (that is, technological infrastructure) yield negligible operational benefits, while the opportunity cost incurred by not procuring additional equipment is significant. Only when the investment is large enough to yield significant operational benefit is it worthwhile.

4. Technological Uncertainty

The R&D process converts expenditures, mainly on the work of scientists and engineers, and the equipment they need, to a new, improved, operational capability. The outcome of this effort is technological progress. This, in turn, serves as an input in the defense production function, whose output is military capability.

Given a certain budget level, it is possible to assess the level of R&D effort it produces with a relatively high degree of certainty. However, the technological progress this effort produces is uncertain. Furthermore, the operational contribution of technological progress is also uncertain, as it depends on external factors. In order to focus on technological uncertainty, the model assumes that the only source of uncertainty is that relating R&D effort to technological progress[4] (the obtained quality level). As discussed in Section 2, we model technological uncertainty as a continuous

[4] It is straightforward, though tedious, to modify the model to account for the uncertainty in the operational contribution of a new technology, that is, uncertainty in the parameters of the S-shaped function.

random variable, allowing *ex-post* technological level to be lower than, equal to or higher than *ex-ante* target levels.

Formally, let $\tilde{q}$ be a random variable representing quality. A risk-neutral defense decision maker chooses a *target* quality level, denoted q, which maximizes his *expected system value*[5]. The choice of the quality target level fully determines the cost of the R&D effort, $c(q)$, and the probability distribution of $\tilde{q}$.

The probability distribution of $\tilde{q}$ is described by its probability density function, denoted $p_q(\tilde{q})$. $p_q(\tilde{q})$ is defined over[6] $[0,\infty]$, and assumed to be continuous and twice differentiable. Furthermore, we define its expected value and standard deviation to be:

Definition (1): $$E(\tilde{q}) \equiv q$$

Definition (2): $$\sigma(\tilde{q}) \equiv \sigma_0 \cdot \left(1 - {g(0)}/{g_{\max}}\right) \cdot q$$

where q is the target quality level, σ_0 is the standard deviation for one unit of R&D effort, assuming no local experience with the technology, g is the S-shaped improvement function and g_{max} is its asymptotic value as q approaches infinity (see Expression 1).

Definition (1) implies that the obtained level of integrative technologies increases with the target level. Intuitively, the more ambitious the R&D effort (and the more costly), the better the outcomes it is expected to yield.

---

[5] It is possible to extend the model to allow for other risk attitudes by incorporating a utility function over the possible realizations of system value.

[6] The range of p represents the assumption that even total failure of an R&D effort cannot result in a negative accumulation of knowledge. At worst, nothing is gained from the R&D effort.

Definition (2) is slightly more complicated: the first term, σ_0, is a measure of the "inherent" uncertainty in developing the new technology[7]; it depends on the type of the technology, and on the existence of any knowledge about it (for example, a technology that is known to exist somewhere else in the world is less uncertain than a completely new technology.) The second term measures prior experience with the technology by comparing the initial value of the technology and its asymptotic value. When there is little experience with a technology, its value is still much smaller than its asymptotic value, and the pace of its future progress is characterized by a relatively high degree of uncertainty. The third term depends on the planned R&D effort; the larger the R&D effort, the greater the uncertainty, and when R&D is done in "small steps" its outcome is more predictable.

In summary, the optimization problem under technological uncertainty is defined by:

$$\begin{aligned} &\max_{q,x} E_{\tilde{q}}\left[V\left(x,\tilde{q}\right)\right] \\ &s.t. \quad \begin{cases} x + c\left(q\right) \le B \\ x, q \ge 0 \end{cases} \end{aligned} \tag{8}$$

where V is the system value function defined in (1). V can be rewritten as:

$$V\left(B-c\left(q\right);\tilde{q}\right) \equiv f\left(B-c\left(q\right)\right)\cdot\left(1+g\left(\tilde{q}\right)\right). \tag{9}$$

The expected value of military capability is then given by:

$$E_{\tilde{q}}\left(V\right) = \int_0^\infty V\left(B-c\left(q\right),\tilde{q}\right) p_q\left(\tilde{q}\right) d\tilde{q} \tag{10}$$

If an internal solution exists, it solves the following first order condition:

---

[7] When σ_0 is zero, the stochastic model converges to the deterministic model.

$$\left.\frac{\partial E_{\tilde{q}}\left[V\left(B-c(q),\tilde{q}\right)\right]}{\partial q}\right|_{q=\hat{q}}=0 \tag{11}$$

where $\hat{q}$ is the optimal target level of q. Finally, Equation (12) provides the first-order condition in an explicit form:

$$\int_0^{\infty}\left[\frac{\partial V\left(B-c(\hat{q}),q\right)}{\partial\hat{q}}\cdot p_{\hat{q}}(q)+V\left(B-c(\hat{q}),q\right)\cdot\frac{\partial p_{\hat{q}}(q)}{\partial\hat{q}}\right]dq=0\,. \tag{12}$$

Solution

The first-order condition (12) cannot be solved analytically. However, it is possible to characterize the solution of (8) by numerical methods. The numerical method used to compute the optimal solution is based on maximizing the integral in (10); this method computes the value of (10) numerically, and finds the value of $\hat{q}$ that maximizes it. Using extensive experimentation, we find, for our model, that this method is significantly faster and more accurate than a Monte-Carlo simulation, and more robust than numerically equating the first-order condition to zero (that is, solving Equation 12).

The numerical solution of the model requires specifying the various functions. Following Hirao (1994), Garcia-Alonso (1999), Setter (2004) and Setter and Tishler (2004), we use $f(x)=x^{\rho}$ (where ρ<1), $g(q)=\frac{1}{\delta_1+\exp\left(\delta_2-\delta_3 q\right)}$, and $c(q)=c\cdot q$. We assume that p is Gamma distributed (since it fits the requirements of Definitions (1) and (2), and is analytically tractable).

Equation (13) provides the probability density function of the Gamma distribution. Equations (14) and (15) give its expected value and standard deviation (Milton and Arnold, 1990).

$$G\left(\tilde{q};\alpha,\beta\right)=\frac{\tilde{q}^{\alpha-1}\exp\left(-\frac{\tilde{q}}{\beta}\right)}{\Gamma\left(\alpha\right)\beta^{\alpha}} \tag{13}$$

where $\Gamma\left(\cdot\right)$ is the Gamma function.

$$E\left(\tilde{q}\right)=\alpha\beta \tag{14}$$

$$\sigma\left(\tilde{q}\right)=\sqrt{\alpha}\beta \tag{15}$$

The parameters of the distribution, α and β, are chosen to yield the required expected value and standard deviation. Figure 1 shows some examples of the Gamma distribution with varying means and standard deviations. The mode of a Gamma distribution is smaller than the expected value; hence, it is more likely that the outcome of an R&D effort will be lower than the expected value. Nevertheless, the distribution has a long right tail, that is, little potential for achieving very high technological levels. The figure also demonstrates the effect of the size of the R&D effort. Greater R&D effort increases both the expected value and the standard deviation of the obtained technological level.

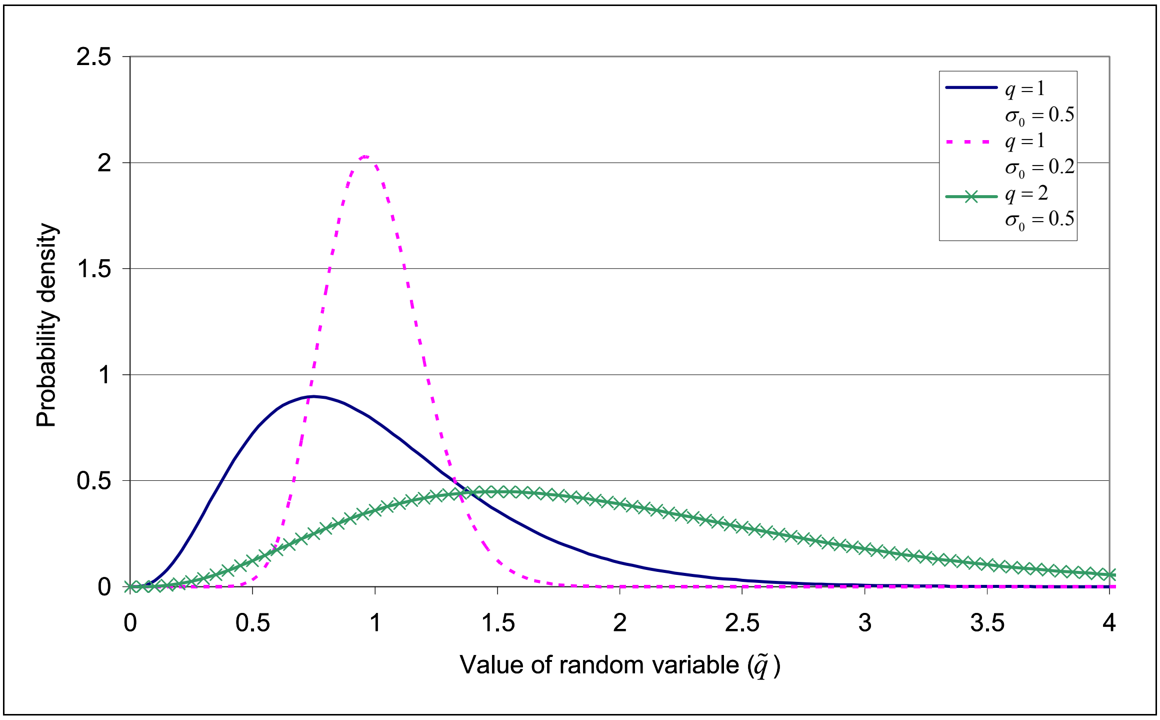

Figure 1: Illustration of the Gamma probability density function

In order to illustrate the optimal solution, we used the following parameter values[8]: ρ=0.5, δ_1=1, δ_2=3, δ_3=1 and c=1. Figure 2 shows the optimal expenditure on q as a function of B, the budget level. The various curves represent different values of σ_0, the "inherent" uncertainty level of the technology. Figure 3 shows the expected value of the weapon system (at the optimal level of $\hat{q}$), divided by V_{Det}, the value of this weapon system in the deterministic model.

[8] These values were chosen to satisfy the budget threshold condition specified in Proposition 2. The nature of the results was maintained for any other set of parameter values we tried that satisfy the budget threshold condition.

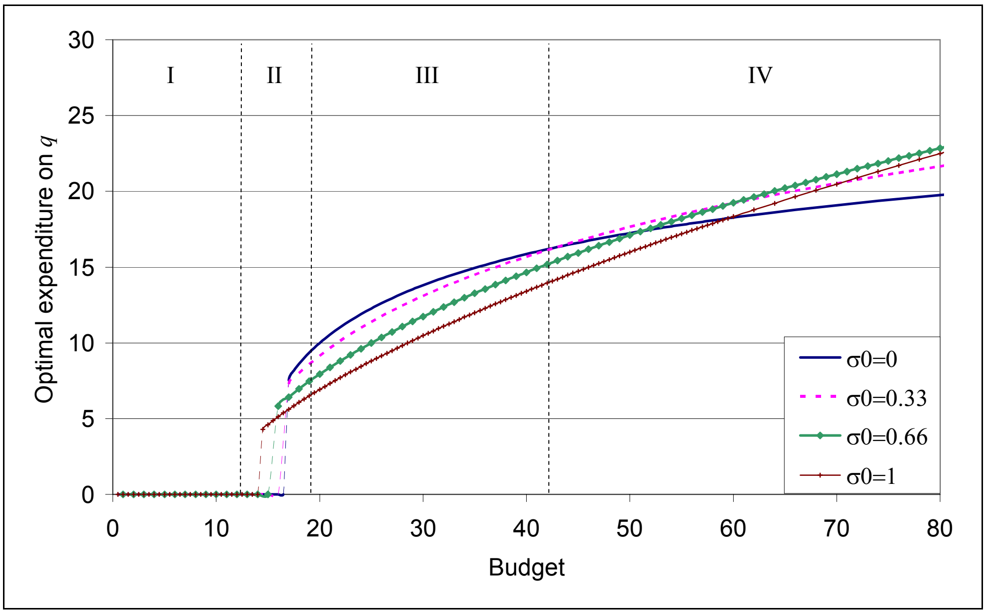

Figure 2: Optimal expenditures for different values of σ_0

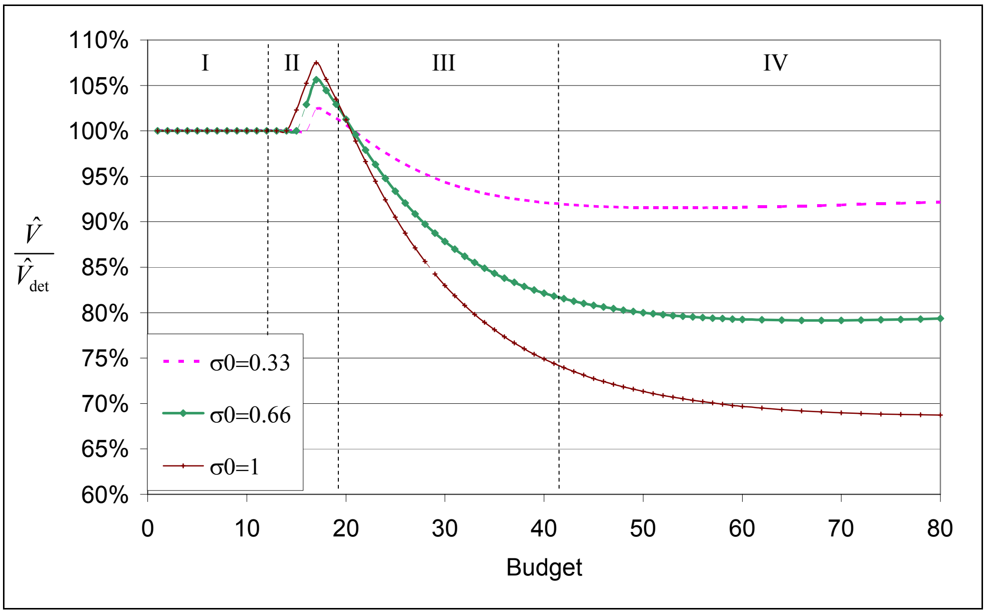

Figure 3: System value, V, for different values of σ_0 (relative to the military capability of the deterministic model – V_{det})

Figure 2 is divided into four segments[9], separated by dotted lines: in segment I, where the budget level is very low (lower than 15 in this example), it is optimal to spend the entire budget on procurement (quantity). This result maintains the intuition of the budget threshold result of Proposition 2: when the budget is small, the optimal expenditure on R&D of advanced technologies is zero.

In segment II, where the budget level is higher, it may become optimal to spend on R&D of advanced technologies, depending on the level of uncertainty. Surprisingly, the budget threshold is *lower* when uncertainty is *higher* (see Figure 2). Furthermore, the obtained expected value is also higher for the high uncertainty cases. Thus, a nation with a budget in this area will tend to devote its R&D expenditures to highly uncertain and novel technologies.

The intuition of this result stems from the possible value of "good luck" in the *convex* early stage of technology: when there is no uncertainty, the attained level of quality yields very small benefits for small budgets – it is still in the flat part of the S curve. As uncertainty grows higher, the probability of reaching the fast-rising part of the S curve is higher. This is evident in Figure 4 showing the probability to achieve a "significant" operational contribution from a certain quality level[10] as a function of σ_0 (the "inherent" uncertainty of the technology). Although high uncertainty also means a higher probability of reaching very low levels of integrative technologies, the

---

[9] Different parameter values yield different optimal values, but the general behavior of the model remains the same for a very wide range of parameter values (as long as the budget threshold condition is met).

[10] There is no specific value of q above which operational contribution is "significant". We chose the value of q at the point where $g(q) = 0.3$, that is, where the technology reaches 30% of its potential value, though the intuition holds for any other value lower than 50%.

downside is limited in its effect on system value, because of the convexity of the improvement function in that area. Thus, in the case of small budgets, the value of "good luck" more than offsets the repercussions of R&D failures, and it is optimal to spend on R&D in the hope of a large success (large realization of $\tilde{q}$).

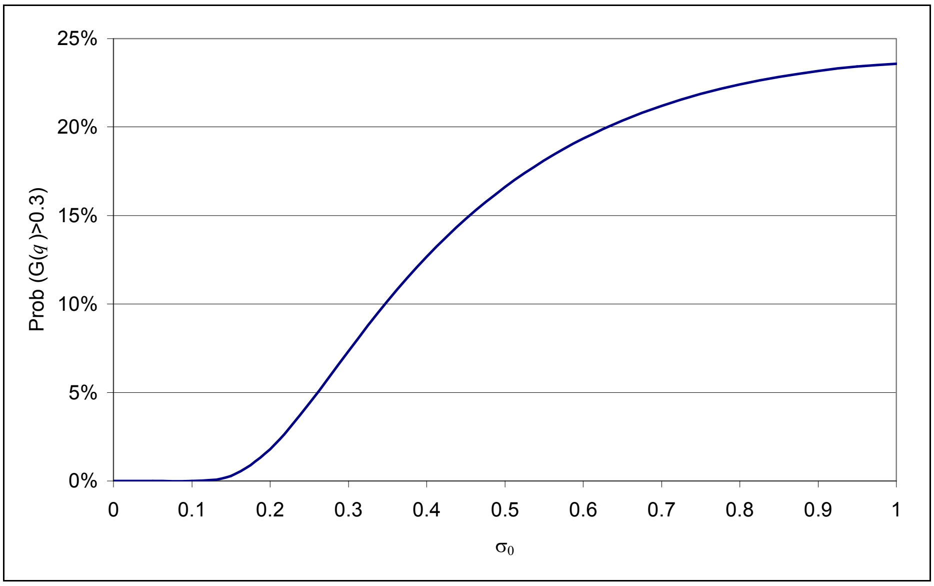

Figure 4: The probability that R&D of integrative technologies will be highly successful as a function of σ_0

The third segment depicts cases with relatively high budget levels, in which it is optimal to spend on R&D for all uncertainty levels. The optimal expenditure is inversely related to uncertainty, and expenditure is maximal for the deterministic case. In this segment, the attained level of advanced technologies for the deterministic case is in the fast-rising part of the S curve (around the inflection point). The contribution of "good luck" is limited in value because of the concavity of the S curve, while the convex downside, in case of failure, is more significant in its influence. This leads to a "risk-averse" attitude, and a preference for low risk technological R&D projects.

When the budget levels are very high, as in segment IV, optimal expenditure on advanced technologies once more becomes higher when uncertainty is higher. The intuition here suggests an opposite behavior to that of the second segment: the expenditure is higher to reduce the possible effect of "bad luck": the expected level of quality is very high; thus the possible upside is very small, in terms of operational effectiveness. On the other hand, the downside might be very damaging, as the S curve is steep to the left of the expected value. By increasing the expenditure on advanced technologies, the expected value and standard deviation increase, but the net effect is still reducing the probability of obtaining a very low realization ("total failure", for example) of the R&D project. Unlike segment II, in this case system value is significantly higher when the uncertainty is lower (see Figure 3). Thus, when it is possible to tackle a technological problem by several R&D programs, each with different uncertainty level, the less risky one should be chosen. However, when considering an investment in a specific technology, more should be invested in the riskier the technology.

A rudimentary comparison of the defense R&D characteristics of some countries supports[11] the results implied in Figure 3: most Middle-Eastern countries have low defense expenditures and a low level of advanced integrative technologies[12] (Gordon, 2003). Israel, despite having a relatively low budget, is known for its technological leadership and indigenous weapon systems. Moreover, some of its self-developed advanced integrative systems are considered world leaders (for example, Yemini, 2003, quoted the commander-in-chief of the Israeli Air Force saying that the

---

[11] This comparison may not explain all the differences in the risk characteristics of defense R&D ventures across countries. There are, of course, other relevant factors: cultural differences (which affect risk attitudes), initial technological level (that is, the S curve parameters), price levels, etc.

[12] In the last two decades it has become customary in defense circles (and the literature) to refer to "integrative" technologies, which integrate the various systems of the military, as the most advanced defense technologies (see Setter, 2004).

IAF's tactical data network is more advanced than that of the American Air Force). This may be explained by a choice of particularly risky R&D projects which produce remarkable results when they succeed.

The UK and some other West European countries fit with the description of the third group: they spend on technology, but are considered relatively conservative in their choice of R&D projects (Barzilay, 2003a). In particular, their integrative technologies are often developed by the US, or jointly with the US (for example, the Joint Strike Fighter program), which could be interpreted as an investment in lower-risk R&D (see, for example, IISS, 2000).

Finally, the US acquisition budget level is an order of magnitude greater than that of any other country in the world. The US also spends significant portions of its budget on highly risky R&D projects, possibly as a hedge against failures. Gansler (1989) addresses this issue explicitly, stating that high-risk areas need to be adequately funded, and that alternatives must be available to cover the likely areas of failure.

5. A Dynamic Model with Technological Uncertainty

Acquisition processes are lengthy, often on the order of 10-15 years (Gansler, 1989). Along such a period of time, decisions regarding investment in new systems may be adapted to reflect the appearance of technological and other types of uncertainty. In this section we explore the effect of decision flexibility on the optimal budget allocation.

We define the intertemporal value function to be a discounted sum of interim system value levels[13]. Formally,

$$V\left(X_1,\ldots,X_T,Q_1,\ldots,Q_T\right)=\sum_{t=1}^{T}\delta^{t-1}V_t\left(X_t,Q_t\right) \tag{16}$$

where $V_t\left(X_t,Q_t\right)$ is the system value at time t (defined according to Equation 1), X_t and Q_t are the stocks of quantity and quality at time t, and δ is the discount factor. Stocks of quantity and quality are accumulated from period to period (representing the accumulation of equipment and the advancement of technology) in accordance with the following difference equation:

$$Y_{t+1}=\left(1-\mu\right)\cdot Y_t+y_{t+1} \tag{17}$$

where Y_t is the stock of either quantity or quality at time t, y_t is the respective flow, and μ is the depreciation rate, representing wear and tear of equipment and obsolescence of technology. In sum, the defense decision maker maximizes V, by choosing the flow of quantity, x_t, and target quality level, $\hat{q}_t$, subject to a per-period budget constraint, B_t.

The level of technological uncertainty in each period is extended directly from definition (2) of the static problem. In essence, we assume that the standard deviation of technology outcome in period t, $\sigma\left(\tilde{q}_t\mid q_1,\ldots,q_{t-1}\right)$, depends only on the resulting stock at the beginning of the period. Specifically,

---

[13] This is not a trivial choice: for example, if the value of the system in the next period is very low, the country will be subject to an attack by its rivals, thus rendering high values in future periods irrelevant. If the system in question is just one of many used by the country, then a discounted sum formulation is adequate. Alternatively, a discount product formulation is also possible (see Setter, 2004, for an example of such a formulation), and can be shown to yield similar results.

$$\sigma\left(\tilde{q}_t \mid q_1,\ldots,q_{t-1}\right) \equiv \sigma_0 \cdot \left(g_{\max} - g\left(Q_{t-1}\right)\right) \cdot \hat{q}_t \tag{18}$$

Hence, *ceteris paribus*, the larger the realization in the first period, the smaller the uncertainty in the second period. For example, if $q_1>0$, then $g(q_1)>g(0)$, and therefore $\sigma\left(\tilde{q}_2 \mid Q_1\right) < \sigma\left(\tilde{q}_1\right)$. If we assume that the probability density function of $\tilde{q}_t$ depends only on its expected value and standard deviation, then Expression (18) provides the required information to obtain the conditional probability density function $p_{\hat{q}_1,\ldots,\hat{q}_t}\left(q_t \mid Q_{t-1}\right)$ and the joint probability density function $p_{\hat{q}_1,\ldots,\hat{q}_T}\left(q_1,\ldots,q_T\right)$.

This budget allocation problem can be solved in advance, providing the optimal budget allocation for each period. Formally, this would entail solving the following program:

$$\hat{V}_{commit} \equiv \max_{\hat{q}_1,\ldots,\hat{q}_T} E\left(V\right) = \max_{\hat{q}_1,\ldots,\hat{q}_T} \int_0^{\infty} \cdots \int_0^{\infty} V \cdot p_{\hat{q}_1,\ldots,\hat{q}_T}\left(q_1,\ldots,q_T\right) dq_T \cdots dq_1 \tag{19}$$

We refer to this model as a "*commitment*" model, since the decision maker commits in advance to a specific allocation of resources.

Alternatively, a decision maker may be better off viewing this R&D effort as a *sequential* decision process (Rogerson, 1995; Roberts and Weitzman, 1981). Instead of committing in advance to future expenditures, such a decision maker will periodically review the outcomes of his past decisions, and will have the flexibility to adjust his future expenditures accordingly. That is, $\hat{q}_2$ is set only after the realization of $\tilde{q}_1$, $\hat{q}_3$ is set only after the realization of $\tilde{q}_2$, and so forth. Thus, in contrast to Expression (19), the objective function of the "*decision flexibility*" model is formally defined by the following dynamic program (Denardo, 1982).

$$\hat{V}_{flex} \equiv \max_{\hat{q}_1} E\left(V_1 + \delta \max_{\hat{q}_2} E\left(V_2 + \delta \max_{\hat{q}_3}\left(\cdots\right) \mid Q_1\right)\right) \tag{20}$$

In this model, the decision maker maximizes the value function by choosing only the first period's expenditure level, taking into account that the second period's expenditure level will be chosen later on, after the outcome of the first period is known. Hence, the optimal solution to this problem entails an optimal value, $\hat{q}_1$, and "reaction" functions $\hat{q}_t\left(Q_{t-1}\right)$, which prescribe the optimal policy for each period's expenditure contingent on the realization of the previous period.

Solution

The model cannot be solved analytically, and requires a numerical dynamic programming solution based on a recursive procedure, which numerically computes and maximizes the expected values. The solution of Expression (20) is demanding computationally; it maximizes an integral, whose integrand is itself a maximization of an integral, and so forth. Hence, computation time grows exponentially with the number of periods. The examples below were therefore solved for two periods. We used the same functions (for *f*, *g* and *p*) and parameters as in Section 3, and assumed: B_1=B_2, δ=0.1, and μ=0.9. While the uniqueness of an optimal solution was not proved, the numerical computation always converged to the same solution, regardless of the initial value.

Figure 5 compares the value of the two models at the optimal solution, as a function of the budget level and the uncertainty level. The figure shows that the ability to adjust the expenditure level has a positive value. The value of flexibility increases with both budget level and uncertainty.

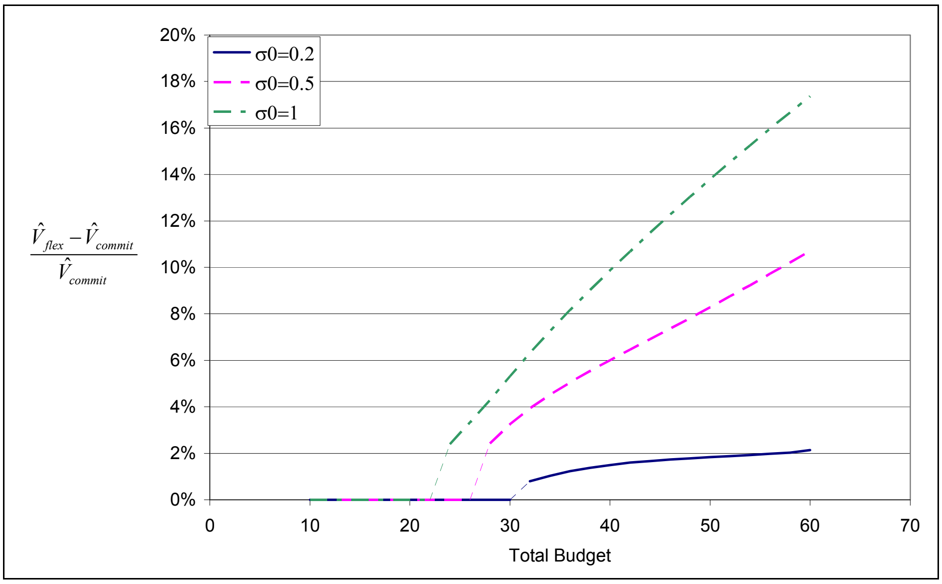

Figure 5: The improvement value achieved by decision flexibility (compared with the commitment model) measured in military capability units.

The value of flexibility in R&D decisions is thoroughly discussed in the "real options" literature (Trigeorgis, 1996; Pennings and Lint, 1997). The concept of a "real option" builds on financial options, which are instruments that provide an investor with the right, but not the obligation, to buy or sell a financial asset (for example, stocks, bonds, currency, etc.), at a given price at some known date in the future. Real investment opportunities often include options of a similar nature. For example, entering an R&D project provides a firm with the option to abandon the project if it is technically unsuccessful, or if market conditions change before market launch. Investment in an R&D project also entitles the investing firm to the option to enjoy the fruits of future generations of the project, which are not planned at the time of the initial decision. A well-established result in the options literature (both financial and real) is that their value increases with uncertainty (Dixit and Pindyck, 1994).

Thus, in the case of defense R&D, the flexibility to adjust future expenditures based on interim technological progress may bear a significant option value. To better understand the sources of this option value, Figure 6 shows an example of a "reaction" function – the optimal decision for the second period conditional on the outcome of the first period. If the first period outcome is very poor, and hardly any technological progress has been made, then it is optimal (in this example) to try again and spend a significant amount on advanced technologies. As the outcome of the first period improves, the optimal level in the second period decreases, until it reaches some point (when the first period R&D is highly successful), at which it is optimal to spend the entire budget on procurement of established systems (that is, $\hat{q}_2 = 0$). The probability of a significant adjustment grows with uncertainty, since extreme values are more likely, explaining the increase of option value with uncertainty.

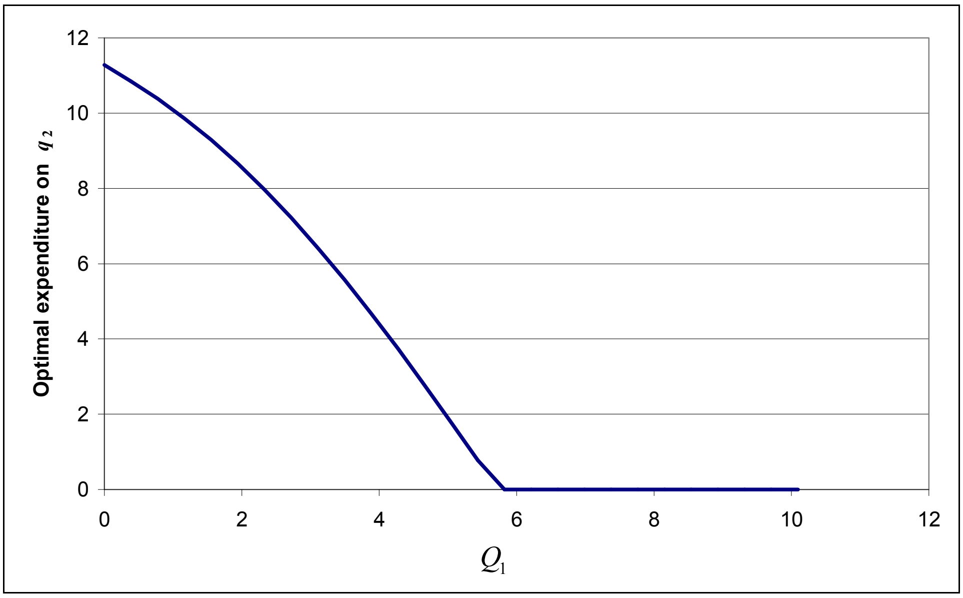

Figure 6: An example of a "reaction" function, that is, the second period's optimal target level as a function of the first period's outcome

Figure 7 compares the first-period optimal decision in the decision flexibility model with the commitment model (for the case of σ_0=1). With decision flexibility, a more "aggressive" approach is optimal in the first period – the expenditure on the uncertain advanced technologies is higher than in the first period of the commitment model. Because of the adjustment option of the second period, higher risks may be taken in the first period. This result stands in contrast to the standard real options result, in which the existence of the real option causes a delay in expenditures, waiting for uncertainty to unfold (Trigeorgis, 1996). The reason for this striking difference is the following: most models take uncertainty to be exogenous (that is, it is resolved by the mere passage of time), while in our model uncertainty is also determined by the first-period decision (see definition 2); the more "ambitious" the R&D effort, the more uncertain its outcome. Since the option value increases with uncertainty, the existence of the adjustment option causes the decision maker to *increase* the uncertainty in the first period, by setting a higher target level.

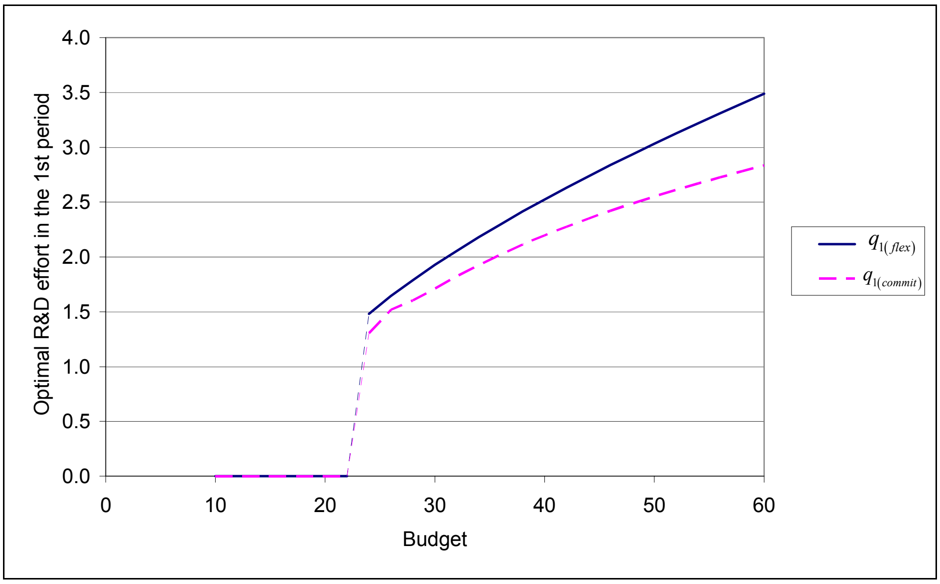

Figure 7: Comparison of the optimal first-period target level of integrative technologies for the cases of commitment, $q_{1(commit)}$, and flexibility, $q_{1(flex)}$.

In summary, the results of this section imply that it may be important to maintain decision flexibility, particularly for large and uncertain R&D projects. The model allows calculating the price worth paying to keep flexibility.

Barzilay (2003b) provides another recent example of the value of flexibility in military procurement: he quotes the IDF's deputy head of the planning department as saying that due to uncertainty in the future battlefield, the IDF must change its resource allocation mechanisms to allow it to shape the future. As an example, he mentions the latest fighter plane procurement transaction in which the Israeli Air Force committed to buying 102 fighters. In his view, it might have been better to first buy 50 fighters, and defer the decision on the additional 52 planes to a later date.

Since flexibility has a positive value, one could expect decisions regarding R&D projects to be taken every year, if not every day. Not only is this not the case, but commitments of much greater timescales are also often made[14]. There are three reasons for this. First, the review and decision process is costly, thus making it not cost effective to adjust decisions too often. Second, commitment may reduce costs – the unit cost of both R&D and procurement may decrease if the government commits to buying a significant quantity in advance. Finally, commitment may have a strategic value (Dixit and Nalebuff, 1993), by affecting decisions of other nations. For example, the commitment of the Reagan administration to the Strategic Defense Initiative (commonly referred to as the "Star Wars") may have caused the Soviet Union to spend on counter-initiatives to the point of national bankruptcy (Koubi, 1999).

---

[14] A prominent example is President Kennedy's commitment in 1960 to put a man on the moon by the end of the decade.

6. Conclusion

Technology is regarded as a key factor in the success of modern organizations, and specifically in that of military organizations. While many technologies enjoy small incremental improvements, it is the R&D of advanced state-of-the-art technologies that shapes the face of the future battlefield. In this paper, we model such advanced technologies by their S-shaped returns profile: they require significant infrastructure investment, followed by a period of increasing returns, before they become mature, and face decreasing marginal benefits.

In a deterministic setup, we provide a closed-form optimal solution to budget the allocation decision of procurement and R&D, and show that the optimal investment in such technologies is discontinuous: when budget levels are below some threshold, it is not optimal to invest in advanced technologies at all. Above the threshold, however, a sizable investment is optimal.

We demonstrate that when technological uncertainty is introduced the optimal behavior is somewhat more complicated: while very low budgets still make investment in advanced R&D prohibitive, somewhat larger budgets require investments in highly uncertain advanced technologies. Such technologies, if successful, may provide their developer with decisive advantages over rivals. Yet higher budgets allow for investment in advanced R&D, but with a preference for low-risk programs: the budget is high enough to ensure "good enough" capability, without the need to take high risks. Finally, for very high budget levels, highly uncertain technologies are allocated a larger share of the budget, to ensure that their development is successful.

We conjecture that this result extends to a business setting. If so, then in such an extended model, start-ups and large firms will tend to invest in risky R&D, while middle-sized firms will opt for lower risks. This conjecture may be tested both theoretically and empirically.

We further show that maintaining the flexibility to adjust investments along the R&D program is beneficial, in accordance with standard "real options" results. We do, however, show that this flexibility may lead decision makers to invest more in earlier periods, in order to enjoy better control over the results of the latter parts of the program.

These results have clear policy implications. First, countries may measure programs along the returns vs. budget size, in order to find the optimal investment policy for each program. Such an investment policy should answer questions like: should one invest in the program? If so, how much should be spent? What development approach should be taken (in terms of uncertainty)?

Second, the trade-off of flexibility and commitment should be dealt with explicitly, taking into account the various motivations for each. Furthermore, once a decision is taken with regard to the level of flexibility to be maintained in an R&D program, the dynamic investment strategy should be adapted accordingly.

7. References

Barilzay, A. (2003a), "Strong disagreements should not interfere with cooperation", Haaretz daily newspaper (in Hebrew), 30.9.03.

Barilzay, A. (2003b), "IDF 2020", Haaretz daily newspaper (in Hebrew), 24.10.03.

Christensen, C. M. (1992), "Exploring the limits of the technology S-curve. Part II: Architectural technologies", *Production and Operations Management*, 1, 358-366.

Dasgupta, P. and E. Maskin (1987), "The simple economics of research portfolios", *The Economic Journal*, 97, 581-595.

Denardo, E.V. (1982), D*ynamic programming: models and applications*, Englewood Cliffs, NJ: Prentice-Hall.

Dixit, A.K. (1987), "Strategic behavior in contests", *The American Economic Review*, 77, 891-898.

Dixit, A.K. and B.J. Nalebuff (1994), *Thinking strategically*, New York, NY: W.W. Norton and Co.

Dixit, A.K. and R.S. Pyndick (1994), *Investment under uncertainty*, Princeton, NJ: Princeton University Press.

DoD (2004), *National defense budget estimates for fiscal year 2003*, Washington D.C.: United States Department of Defense.

Gansler, J.S. (1989), *Affording defense*, Cambridge, MA: MIT Press.

Garcia-Alonso, M.C. (1999), "Price competition in a model of arms trade", *Defence and Peace Economics*, 10, 273-303.

Goolsbee, A. (1998), "Does government R&D policy mainly benefit scientists and engineers?", *The American Economic Review*, 88, 298-302.

Gordon, S.L. (2003), *Dimensions of quality*, Tel Aviv: Jaffe Center for Strategic Studies.

Grossman G.M. and C. Shapiro (1986), "Optimal dynamic R&D programs", *The Rand Journal of Economics*, 17, 581-593.

Hirao, Y. (1994), "Quality versus quantity in arms races", *Southern Economic Journal*, 2, 96-103.

IISS (2000), *The military balance 2000-2001*, International Institute for Strategic Studies.

Koubi, V. (1999), "Military technology races", *International Organization*, 53, 537-565.

Loch, C.H. and S. Kavadias, (2002), "Dynamic portfolio selection of NPD programs using marginal returns", *Management Science*, 28, 1227-1241.

Milton, J.S. and J.C. Arnold (1990), *Introduction to probability theory - 2nd edition*, New York, NY: McGraw-Hill Publishing Company

Pennings, E. and O. Lint (1997), "The option value of advanced R&D", *European Journal of Operational Research*, 103, 83-94.

Roberts, K. and M.L. Weitzman (1981), "Funding criteria for research, development, and exploration projects", *Econometrica*, 49, 1261-1288.

Rogerson, W.P. (1995), "Incentive models of the defense procurement process", in K. Hartley and T. Sandler (Eds.), *Handbook of defense economics: Volume I*, Amsterdam: Elsevier Science B.V.

Setter (2004), "Defense R&D in the information age: Analysis of budget allocation decisions", unpublished doctoral dissertation, Tel Aviv University.

Setter and Tishler (2004), "Budget allocation for integrative technologies: Theory and application to the US military", Tel Aviv University, mimeo.

Trigeorgis, L. (1996), *Real options*, Cambridge, MA: MIT Press

Yemini, G. (2003), "The Army is a decade behind the Air Force in computing and automation", Haaretz daily newspaper (in Hebrew), 17.11.2003.

Appendix

Proof of Proposition 1:

The proof is structured as follows: first, the proof shows the conditions for which Equation (6) obtains an internal solution q^*. Second, we use q^* to prove the necessary and sufficient condition (5) for the existence of an internal solution to program (1). Finally we prove the uniqueness of the solution. The global optimum condition is trivial.

MC(*q*) is strictly positive and increasing for all *q* (that is, *MC*'(*q*)>0). $\lim_{q \to B} MC(q) = \infty$.

MB(*q*) is decreasing (has a negative derivative) for $q > \tilde{q}$. Assuming *MB*'(0)>*MC*'(0), then there is a point, q^*, for which *MB*'(*q*)=*MC*'(*q*). This point maximizes *MB*(*q*)-*MC*(*q*).

It is straightforward to show that *MB*(*B*)<*MC*(*B*). An optimal internal solution exists if, and only if, there is a *q* for which *MB*(*q*)>*MC*(*q*). If $MB(q^*)$-$MC(q^*)$>0, then an optimal solution exists. If, however, $MB(q^*)$-$MC(q^*)$<0, then there is no *q* for which this difference is positive, because q^* maximizes *MB*(*q*)-*MC*(*q*). Hence, condition (5) is a necessary and sufficient existence condition.

We prove the uniqueness of the optimal solution by examining two cases:

1. $\hat{q} > \tilde{q}$: since $MB'(\tilde{q}) = 0$ and is decreasing afterwards and $MC'(q) > 0$ then they cannot be equal again. Formally:

$$\forall q > \hat{q}, MB(q) - MC(q) = \int_{\hat{q}}^{q} \underbrace{\left[\underbrace{MB'(x)}_{\leq 0} - \underbrace{MC'(x)}_{>0} \right]}_{<0} dx < 0$$

2. $\hat{q} < \tilde{q}$: Assume there is another $\hat{q}_2 > \hat{q}$ for which *MB*=*MC*. it is easy to show that at $\hat{q}$, (MB-MC)'<0, thus there is a *q* for which $MB(q) < MC(q)$. In order for $\hat{q}_2$ to exist, there must be some *q* for which $(MB(q) - MC(q))' > 0$, and specifically, a *q* for which $(MB(q) - MC(q))' = 0$. This is in contradiction to the uniqueness of q^*

We prove it is a maximum by using the first derivative rule for a maximum: we know that at q^*, $MB(q^*) > MC(q^*) \Rightarrow (\ln V)' > 0$, and at $q = c^{-1}(B)$, $MB(q) < MC(q) \Rightarrow (\ln V)' < 0$ after. Because of the uniqueness of the solution in the interval, these hold in the neighborhood of $\hat{q}$ and using the first rule, this is a maximum. *Q.E.D.*

Proof of Proposition 2:

MB(*q*) is independent of *B*. As for MC(q), recall that $\tilde{f}(q) = f(B - c(q))$. Hence:

$$\frac{\partial MC(q)}{\partial B} = -\left(\frac{\tilde{f}'}{\tilde{f}}\right)_B = -\left[\underbrace{\frac{\tilde{f}''}{\tilde{f}}}_{>0} - \underbrace{\frac{\tilde{f}'f'}{\tilde{f}^2}}_{<0}\right] < 0$$

Thus, the smaller the budget, the higher the value of *MC*(*q*). Since we showed that MB(q) is decreasing above $\tilde{q}$, it is maximized for some $q < \tilde{q}$. For small enough budget, $MC(0) > \max_q MB(q)$. *MC*(*q*) is increasing with *q*, so there is a small enough budget for which condition (5) cannot hold. As *B* increases, *MC*(*q*) decreases monotonously, and thus there is a budget threshold B, for which condition (5) is met, and in which $\hat{q} > q^* > 0$. *Q.E.D.*

Working and Position Papers

1) Peled D, Ben-Haim Y, Ben-Gad M.: "Allocating Security Expenditures under Knightian Uncertainty: an Info-Gap Approach "Economy of National Security Program (ENS) Working Papers Series ENS-WP-1 February 2007.

2) Amiram Oren and Zalman F. Shiffer: The Economic Consequences of The Use and Control of Land Resources by the Defense Sector in Israel" Economy of National Security Program (ENS) Working Papers Series ENS-WP-2, 15 June 2007

3) Paikowsky D. , " Space Technology, Patterns of Warfare and Force Build-up: Between a Power and a Small State" Economy of National Security Program (ENS) Working Papers Series ENS-WP-3, June 2007.

4) Berrebi, Claude and Klor, Esteban F.: "Are Voters Sensitive to Terrorism? Direct Evidence from the Israeli Electorate," Economy of National Security Program (ENS) Working Paper Series ENS-WP-4, July 2007.

5) David A. Jaeger and M. Daniele Paserman, : " The Cycle of Violence? An Empirical Analysis of Fatalities in the Palestinian-Israeli Conflict ", Economy of National Security Program (ENS) Working Paper Series ENS-WP-5, July 2007.

6) Oren Setter and Asher Tishler, Tel Aviv University, "Investment Policies in Defense R&D Programs15", Economy of National Security Program (ENS) Working Paper Series ENS-WP-6, October 2007.

---

[15] We are grateful to Sarit Markovich and to seminar participants in Tel Aviv University and the University of Iowa for their helpful comments. This research was supported by the Economics of National Security Program, Samuel Neaman Institute.

Dr. Oren Setter serves as Head of Operations Research branch at the Directorate of Defense Research and Development (MAFAT), Israeli Ministry of Defense. Dr. Setter has a Ph.D in Operations Research from Tel-Aviv University and he received his B.Sc in Physics from the Hebrew University (as participant of the Talpiot Program). Dr. Setter is an adjunct lecturer within the Faculty of Management at Tel Aviv University, and published several scholarly articles and book chapters.

Professor Asher Tishler received his B.A degree in Economics and Statistics from the Hebrew University of Jeruslalem in 1972, and a Ph.D in Economics from the University of Pennsylvania in 1976. Since 1976 he is a faculty member at the Faculty of Management at Tel Aviv University. Professor Tishler serves as the director of both BRM Institute for Society and Technology and the Eli Hurvitz Institute for Strategic Management at Tel Aviv University. He was a visiting Professor at the University of Southern California, University of Iowa, and University of Pennsylvania. Professor Tishler's research interests include: Defense Economics, Economics of Energy, Research and Development and Research Methodologies. He has published more than ninety papers in professional journals, and serves as an advisor to several companies in Israel and abroad.

The ENS Program, established in late 2003, is an inter-mural program aiming to initiate, encourage, and facilitate high quality academic research and policy position papers on the interconnections between economics and defense. The close links between economic strength and development on one hand, and defense capabilities and security on the other are well recognized. Nevertheless, there is little theoretical and empirical research on these links by the academic community in Israel available to support policy making in these critically important matters. The Program holds periodic research meetings, organizes workshops on defense economics, and provides financial support on a competitive basis to proposals by researchers and graduate students submitted in response to widely circulated Calls for Proposals. Program participants include economists and researchers in other disciplines from various universities in Israel, research departments in the Bank of Israel and other government agencies, and some current and past officials in government and defense related organizations and industries. The Program Director is Prof. Dan Peled and the Coordinator is Col. (Res.) Moshe Elad.

Samuel Neaman Institute
for Advanced Studies in Science and Technology
Technion-Israel Institute of Technology
Technion City, Haifa 32000, Israel
Tel: 04-8292329, Fax: 04-8231889
www.neaman.org.il