



# A Micro-Economic Approach to Government Support of R&D Investments in the Private Sector

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**Investments in the Private Sector** 

**ABSTRACT** 

This paper focuses on the economic decisions of the government to subsidize investments

in the private sector, and to discriminate among firms in its support programs. The presumption is

that, by taxing corporate profits the government, affects investment decisions made by

corporations and causes them to invest less than what would be socially optimal. Accordingly,

investments that are desirable from the standpoint of social welfare may be rejected by

shareholders. Rejecting investment may ultimately lead to the collection of fewer taxes. We

analyze the conditions for optimal subsidies for investments carried out by the private sector. We

find that high-risk ventures that generate substantial spill-over activity are prime candidates for

government incentive schemes. Programs in which the government provides investment grants

and charges royalties from future revenues are evaluated. The primary advantage of investment

subsidies over tax cuts is the flexibility it renders governments, enabling them to discriminate,

among firms, industries or corporate activity, such as industrial Research and Development.

Keywords: corporate tax, investment subsidies, social present value, externalities.

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#### I. Introduction

In this paper we conduct a theoretical analysis of government incentives for private investment activity. We focus on two major questions: First, under what conditions is a government justified, from an economic viewpoint, to support investment activities undertaken by private firms? Second, under which conditions is support of an investment in R&D projects more economically justifiable than support of an investment in a traditional low-tech project?

In our analysis, we focus on the micro-economic interaction between the government and a specific firm. The approach is based on a partial equilibrium solution, within the framework of Modigliani &Miller's (1958, 1963) (M&M) modeling of corporate valuation. It is assumed that there are no taxes save corporate taxes, and that within this framework all markets are in equilibrium. In the M&M spirit, governments can "negotiate" with firms in order to maximize the total net value to all stakeholders.

The government is viewed as an economic agent that collects taxes from corporations based on simple, uniform tax rules, and provides a basket of services to the entire population. Through its traditional tax rules, governments may affect and alter the investment decisions of the firm, causing a distortion in the allocation of resources throughout the economy. We examine the conditions under which the government may provide investment subsidies to an individual company in order to generate economic benefits and enhance public welfare.

The corporate tax rate in most countries is uniform for all corporations. As a result governments often find it easier to introduce differential subsidy schemes, on a case by case basis, rather then to adjust the tax code. An analysis of such schemes and their impact on firms can be found in Levy and Terleckyj (1983), Mansfield and Switzer (1984), Wallsten (2000), Lach

(2001).<sup>1</sup> An international survey of governmental support schemes for R&D is included in a policy paper published by the Canadian Department of Finance and Revenue,<sup>2</sup> with a special emphasis on the G-7 countries and Canada. It has been shown empirically by Mansfield (1986) that for the United States, Canada, and Sweden, tax credits for R&D reduces government revenues by substantially more than the amount added to the entrepreneur. Mansfield concludes, therefore, that tax credits for R&D activity are inefficient. In this paper the welfare effect of subsidies extended differentially to specific projects rather than an across-the-board tax break is analyzed. We find that subsidizing industrial R&D is not necessarily inefficient and can benefit both individual firms, industries and the economy as a whole.

We adopt a micro-economic approach, looking mainly at the firm's level and modeling the decision-making process for corporate investments and how it affects social welfare. We do not presume to prove an optimal corporate tax schedule that eliminates or minimizes tax distortions, rather, we take the tax regime as given: corporate taxes are levied at a uniformed rate on accounting profits. Taxation affects the investment opportunity function in two fundamental ways (as will be shown in Section II): it reduces the NPV of the firm for any investment scope, and, it reduces the firm's optimal level of investment relative to a no-tax scenario.<sup>3</sup> Accordingly, one can identify three general cases in which mitigation of corporate taxation can be justified.

In the first case, corporate taxation may cause the firm to reject projects with a positive pre-tax NPV. By rejecting the project, the firm loses the incremental NPV generated and the

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<sup>&</sup>lt;sup>1</sup> In many countries, e. g. Ireland and Israel special government units are established to analyze all subsidy applications. Subsidies are granted on a case by case basis, based on some general guidelines.

<sup>&</sup>lt;sup>2</sup> Why and how governments support research and development, December 1997. See http://www.fin.gc.ca
<sup>3</sup> For a discussion of this issue, see, for example, Stiglitz (1973), Asimakopulos and Burbidge (1975), and Galai (1998). See also Brealey, et al (1998). Stiglitz (1976) shows how tax can be non-distortive, however he concludes: "True economic depreciation and immediate write-off of capital expenditures are both depreciation policies which are equitable and efficient ... Neither, however, is generally used; actual depreciation allowances introduce elements of inefficiency as well as inequity".

government is denied the tax benefits. In these instances, (discussed in detail in Section III,) the government may be justified to provide subsidies so that the project is accepted.

A second justification for subsidizing investments is found in cases where positive externalities to a specific investment can be identified.<sup>4</sup> A given private investment may positively impact its environment even though the individual firm is unable to capture the economic value of spillover effects. In this case, the government may intervene in order to generate spillovers to other sectors in the economy and encourage entrepreneurs to increase investment to a socially optimal level.<sup>5</sup> This scenario is analyzed in Section V.

A third reason for governments to forego a portion of potential tax revenues is in order to attain a socially optimal level of investment and avoid under-investment. The assumption is that at the socially optimal level of investment the value added to the wealth of the entire society is maximized, benefiting both the firm and government. We define the social benefit of a project by its pre-tax NPV, (and, in the externality case, also by the addition to NPVs of other projects being affected by the specific project). With taxes, the social benefit is measured by the sum of the NPV of the project and the PV of taxes collected by the government. This is a novel approach to the measurement of social welfare in an uncertain environment, where uncertain future social values are converted to present value terms, on an ex ante basis. This differs from the analysis of social costs based on actual ex-post results.

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<sup>&</sup>lt;sup>4</sup> See, for example, Griliches (1979), (1992), and (1995), Jones (1995), Levin and Reiss (1988), Mairesse and Mohen (1995), and Park (1996). Romer (1986) introduces knowledge as an input in the production function that has increasing marginal productivity through externalities.

<sup>&</sup>lt;sup>5</sup> In the report of the Department of Finance and Revenues, Canada, it says: "The economic rationale for governments to assist R&D is that the benefits of R&D spill over, or extend beyond the performers themselves, to other firms and sectors of the economy and the value of these benefits is not fully appropriable by the R&D performer. These "spillover benefits" mean that, in the absence of government support, firms would perform less R&D than is desirable from the economy's point of view. Markets fail to allocate an efficient or socially optimal quantity of resources to the performance of R&D."

It should be noted that while our starting point is the under-investment caused by corporate taxation, other authors focused on the over-investment incentives caused by the limited liability feature of the corporation. John, Senbet and Sundaram (1994) show that corporate taxation can mitigate the risk-taking and over-investment incentives of shareholders in a levered firm. In our paper the firm is assumed to be a pure equity firm.

Obviously, the NPV approach is complicated when the assumption of homogeneous expectations is removed. Laffont and Tirole (1993) study government investment incentive schemes devised to enhance social welfare. They review models in which entrepreneurs and regulators hold different assessments of risk. Regulation is one way through which governments "negotiate" with individual firms on specific issues related to social welfare. Section IV deals with problems encountered when the entrepreneurs' valuation of the project made is considered by the government to be too optimistic by the government and subsidies are withheld.

The paper does not deal with the optimal subsidy or tax policy of the government. The starting point of our analysis is a discriminatory subsidy scheme in which the government provides grants for individual investment projects. Conventional flat-rate corporate taxes continue to be imposed on the firm's accounting profits. The subsidy is conditional and requires that, if the project is successful, the firm repays the government royalties k from future cashflow. We limit our analysis to a one period framework. In this framework, with a flat tax rate and fixed government budget, it is shown in Section VI that a policy of subsidizing high-tech firms rather than low-tech firms is rational. This contention is based on the observation that hi-tech firms typically incorporate two major characteristics that in tandem render them prime candidates for government-induced incentives:

• The risk-return profile of high-tech ventures leads to a more frequent occurrence of sub-optimal project rejection.

• There is evidence that hi-tech firms generate a greater spillover effect that impacts positively on activity in other sectors of the economy.

## II. Investment Decision by an Individual Firm

We assume a company considering an investment in a project of I dollars at time 0. The company expects the project to generate a net cashflow of K(I) dollars at time 1 with a risk neutral probability<sup>6</sup> of  $\pi$ , and 0 dollars with a (risk-neutral) probability of  $1-\pi$ . The firm faces an S-shaped distribution of expected revenues with increasing, constant and decreasing marginal return as the level of investment increases.<sup>7</sup> The corporate tax rate is  $\tau$  and it is applied to the profit from the project K(I)-I. The firm has to determine the optimal size of the investment.<sup>8</sup> The NPV of the investment is given by:

(1) 
$$NPV = -I + \{(K(I) - I)(1 - \tau) + I\}\pi e^{-r}$$

$$= -I + (K(I)(1-\tau) + \tau I)\pi e^{-r}$$

where r is the continuously compounded risk-free rate for one period.  $^9$  If NPV < 0, the firm should reject the project. Suppose that without taxes the firm would have accepted the same project, i. e.

<sup>&</sup>lt;sup>6</sup> See Harrison and Kreps (1979) for a discussion of risk-neutral probabilities. These are adjusted probabilities (for risk tolerance) in that the value of an uncertain project is equal to the expected payoff under this probability, discounted at the risk free rate.

<sup>&</sup>lt;sup>7</sup> We model this by a logistic function, specified below.

<sup>&</sup>lt;sup>8</sup> For simplicity of analysis we assume that the risk neutral probability  $\pi$  does not vary with investment size.

 $<sup>^{9}</sup>$  If we assume the tax code is of the full-loss offset tax type, than expression (1) should be changed to reflect the fact that the tax subsidy  $\tau I$  is certain. Hence, the expression for NPV becomes

<sup>(1&#</sup>x27;)  $NPV = -I + K(I)(1-\tau)\pi e^{-r} + \tau I e^{-r} = -I(1-\tau e^{-r}) + K(I)(1-\tau)\pi e^{-r}.$ 

$$NPV(\tau=0) = -I + K(I) \pi e^{-r} > 0$$

It is assumed henceforth that NPV( $\tau$ =0) > 0, and without taxes and subsidies the firm will opt to invest in the project. By imposing the tax, if NPV<0, the firm rejects the project, and therefore, the government cannot receive tax revenues from it.

The optimal investment decision for the no tax case is I\*, such that

$$K'(I^*) = \frac{1}{\pi e^{-r}}$$

For the tax case, the optimal investment  $I^{**}$  is achieved when:

$$K'(I^{**}) = \frac{1 - \tau \pi e^{-r}}{1 - \tau} \frac{1}{\pi e^{-r}}.$$

Figure 1 depicts the optimal investments for the tax and no tax cases<sup>10</sup>:

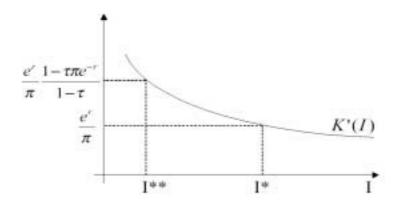


Figure 1. The marginal cash flow as a function of investment size and the optimal investment for the no tax case, I\*, and the tax case I\*\*.

<sup>&</sup>lt;sup>10</sup> Only the relevant part of decreasing economies of scale is shown in Figure 1.

The distortion caused by the existing corporate tax can be corrected by different policies. One approach, for example, is to impose corporate taxes on cash flows (similar in many respects to value added tax) rather than accounting profits. In this paper we focus on an alternative scheme that has been employed to support corporate investments, especially in industrial R&D. This scheme provides government funding for industrial R&D, that is either partially or fully repayable in the future, should the investment prove successful. Repayment can take several forms, such as royalties on gross revenues or profits.

Let us assume that the government is willing to incur a proportion  $\alpha$  of the investment in the project. In return, the government requires a payment of proportion  $\beta$  of I at time 1 only if the project is successful<sup>13</sup>. Note that when  $\beta=0$ , we deal with investment tax credit case; when  $\beta=\alpha$ , if the project is a success the firm returns the initial grant in nominal terms. In general for  $\alpha>0$  and  $\beta>0$  this is similar to a government loan, where  $\frac{\beta}{\alpha}-1$  is the nominal interest and it is paid only in a good state.

The NPV of the projects, given the above subsidy structure, denoted by  $NPV^S$ , and is given by

(2) 
$$NPV^{S} = -(1-\alpha)I + \{(K(I) - \beta I - (1-\alpha)I)(1-\tau) + (1-\alpha)I\}\pi e^{-r}$$
$$= -(1-\alpha)I + \{[K(I) - \beta I] (1-\tau) + (1-\alpha)I\tau\}\pi e^{-r}$$

<sup>12</sup> Hall (1992) reviews the R&D tax policy during the eighties. Trajtenberg (2001) reviews the recent R&D policy in Israel.

<sup>11</sup> See, for example, Galai (1998), and Stiglitz (1976).

 $<sup>^{13}</sup>$  In (2) we assume that the payback to the government is proportional to the initial investment I. An alternative procedure is based on the pay-back on the revenues K(I) up to a ceiling, cI, which is a function of the original investment:

By determining  $\alpha$  and  $\beta$ , given the other parameters, the government can change the decision of the firm from initially rejecting the project (when NPV < 0) to accepting it if NPV^S > 0.

By determining  $\alpha$  and  $\beta$ , given the other parameters, the government can change the decision of the firm from initially rejecting the project (when NPV < 0) to accepting it if NPV<sup>S</sup> > 0. We denote the optimal I where NPV<sup>S</sup> reaches its maximum by I\*\*\*.

Let us denote the amount by which the NPV in the tax case is less than 0 by d:

$$d(I) \equiv - NPV(I)$$
.

Let us denote the net government's grant by g:

(3) 
$$g(I) \equiv \alpha I - \left[\beta I(1-\tau) + \alpha I\tau\right] \pi e^{-r}$$

g denotes the PV of the effective subsidy, where  $\alpha I$  represents the investment grant, and the term in the brackets is the contingent repayment to the government net taxes. When d>0 and there are no subsidies, the investment is rejected. However if for a given I, d>0 (i.e. the NPV is negative), and g>d, then the investment is accepted. Next section deals with optimality considerations for both the entrepreneur and the government.

## III. Why Should Government Subsidize Investments? 14

Having determined the level of subsidies required to reverse the firm's rejection of an economically viable project, we now analyze the conditions governing the government's decision

cI, otherwise.

 $<sup>\</sup>beta K(I)$ , for  $\beta K(I) \le cI$ 

<sup>&</sup>lt;sup>14</sup> The practice of subsidizing R&D projects is discussed in Butler and Mitchel (1998), Irwin and Klenow (1994), and Lichtenberg (1987).

to provide investment grants. The government has to pay g (and  $\alpha I$  at time 0) in order to give the firm an incentive to invest. In return it expects to collect corporate taxes, royalties, and acquire other social benefits.

The government has an incentive to subsidize the marginal investment only if the direct net taxes it collects, minus the subsidy, is sufficiently large, assuming at this stage that the value of externalities is zero<sup>15</sup>. The present value of the contingent tax collection, from the accepted project, is<sup>16</sup>

(4) 
$$G = \tau [K(I) - I]\pi e^{-r}$$

The net present value of the government claim with subsidy is G<sup>S</sup> defined by:

(5) 
$$G^{S}(I) = G(I) - g(I) = -\alpha I + \{ [K(I) - \beta I - (1-\alpha)I] \tau + \beta I \} \pi e^{-r}$$

If prior to the subsidy,  $d(I^{**})>0$ , the firm will reject the project even if it is socially desirable, i.e.  $NPV_{\tau=0}(I^{*})>0$ . However, if a subsidy scheme can be devised such that for  $I^{***}$ 

where  $I^{***}$  is the optimal decision of the corporation given the tax and subsidy schemes, the government is justified, from a purely economic standpoint, to subsidize the investment.<sup>17</sup> If at this point g is greater than d, the company will accept the project. The second condition G > g states that at this point the government has net receipts from the accepted project.

<sup>16</sup> The assumption is that the same risk neutral probability applies to the government's tax claim as to the firm's present value. This assumption is discussed in Section IV.

<sup>&</sup>lt;sup>15</sup> Mayshar (1977) constructs an economic model and investigates the effect of government subsidies on the decision of a risky firm.

<sup>&</sup>lt;sup>17</sup> In Section V we show that in the case of externalities it may be that even if G<sup>s</sup><0 (i.e. G<g), the government may have an incentive to subsidize the investment.

Let us define, H, the "added value to the economy" derived from investing in a project.

This added value to the economy is defined in present value terms, as the sum of the present value accrued to the entrepreneur plus the present value of government receipts:

$$H^S = NPV^S + G^S$$

This term measures the total net value added to the economy by accepting the investment. It takes into account the risk of the projects (through the risk neutral probabilities), as well as the negative and positive effects of taxation and subsidies on the willingness of shareholders to undertake the investment. Governments should remove obstacles that prevent the entrepreneur from accepting a project in which H<sup>S</sup>>0.

Let us assume that the production function is S-shaped, belonging to the class of logistic functions, as follows:

(6) 
$$K(I) = \frac{A_1}{1 + e^{a - bI}} - A_2$$

where  $A_1$ ,  $A_2$ , a and b are constant parameters. By substituting (6) in equations (1) and (2) we express NPV as a function of  $I^{18}$  Equation (2) can be analyzed for the optimal investment decision for three basic cases:

- (1) There are no taxes and no subsidies ( $\tau = \alpha = \beta = 0$ )
- (2) There are corporate taxes but no subsidies  $(\tau > 0, \alpha = \beta = 0)$

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<sup>&</sup>lt;sup>18</sup> See appendix A for a numerical example of (6). The production function exhibit increasing returns to scale for low investments, constant return at medium size investment and decreasing return to scale for higher investment levels.

(3) There are corporate taxes  $\tau$ , subsidies  $\alpha$ , and royalties  $\beta$ , respectively <sup>19</sup>

The partial derivative of (2) is

(7) 
$$\frac{\partial NPV^{S}}{\partial I} = -(1-\alpha) + (1-\tau) [K'(I) - \beta] \pi e^{-r} + (1-\alpha)\tau \pi e^{-r} = (1-\alpha)(\tau \pi e^{-r} - 1) + (1-\tau)\pi e^{-r} [K'(I) - \beta].$$

By equating (7) to zero, we obtain the necessary condition for an optimal investment decision:

(8) 
$$K'(I) = \beta + \frac{(1 - \tau \pi e^{-r})(1 - \alpha)}{(1 - \tau)\pi e^{-r}}$$

where K'(I) is the derivative of K with respect to I.

Equation (8) can also be rewritten in the following way:

(8') 
$$K'(I) = \frac{(1 - \tau \pi e^{-r})}{(1 - \tau)\pi e^{-r}} + \frac{\beta (1 - \tau)\pi e^{-r} - \alpha (1 - \tau \pi e^{-r})}{(1 - \tau)\pi e^{-r}}$$

The first term of (8') is the traditional result of M and M Proposition III on the cost of capital for the tax case.<sup>20</sup> Equation (8') is an extension of the Proposition for the case of government subsidy, which is reflected in the second term.

Equation (8') implies that, for any given project, the investment grant can be fully offset by the contingent royalties since only the linear combination of  $\alpha$  and  $\beta$  matters. Thus, as soon as an increase in  $\alpha$  is accompanied by an appropriate increase in  $\beta$ , the optimal investment decision remains constant.

By assuming that  $\alpha=0$  and  $\beta<0$  we can create a contingent future government subsidy depending on the success of

If for each increase of the initial subsidy  $\alpha$  by an amount  $\Delta\alpha$  the conditional royalty  $\beta$  is increased by  $\Delta\beta$  according to

(9) 
$$\frac{\Delta \beta}{\Delta \alpha} = \frac{1 - e^{-r} \pi \tau}{e^{-r} \pi (1 - \tau)}$$

the NPV<sup>S</sup> function will not change. Hence the optimal decision of the firm will remain intact. As can be seen from (9) the substitution between  $\alpha$  and  $\beta$  is risk class specific, for all projects with the same risk-neutral probability,  $\pi$ .

The optimal investment for the no-tax case, I\*, satisfies

(10) 
$$\frac{bA_1e^{a-bI^*}}{(1+e^{a-bI^*})^2} = \frac{1}{\pi e^{-r}}$$

and the necessary condition for this optimal investment is that  $b\pi e^{-r}A_1 > 4$ .

An analytical expression can be derived when this necessary condition is satisfied. We provide a numerical example in Appendix A for the following parameters:

$$K(I) = \frac{350}{1 + e^{3 - 0.04I}} - 65, \ \pi = 0.6, \ r = 6\%$$

We find that the optimal investment is \$118.7 with a positive NPV=\$13. Hence, with no government intervention the firm will accept the project.

As a result of taxes the highest NPV is achieved at a lower level of investment than in the no-tax case. The investor will require a higher before-tax marginal return in order to compensate

the project.

<sup>&</sup>lt;sup>20</sup> See for example Brealey and Myers (1996, ch. 19).

for the tax. In our numerical example, assuming a 40% tax rate ( $\tau = 40\%$ ), maximum profit is reached at I=\$108.8, but, since the project yields a negative NPV (-12) it will be rejected. Thus both government and the firm forego the opportunity for an added value.

The government should provide incentives to the firm to accept the project as long as the PV of government net receipts,  $G^S$ , is positive. Equation (8) describes the optimality conditions with taxes and subsidies. With  $\alpha$ =0.3 and  $\beta$ =0.2 the optimal investment stands at \$118.1 and the NPV of the project is NPV<sup>S</sup>=\$6.7 and, therefore, should be accepted. The PV of government revenues is  $G^S$ =\$6.3.

This example shows how in the case of a single company the government can induce the firm to undertake a project profitable for both the company and the government by mitigating taxes with subsidies. Without the subsidies, this project does not benefit the society since the firm rejects it. With a combination of taxes and subsidies, however, the firm and the government share both risk and profits.

### IV. The Case of Asymmetric Valuation by Entrepreneurs and Government

So far we have assumed symmetric expectations between the entrepreneur and the government in a sense that both calculate NPV on the basis of the same probability distribution of K(I), and hence the same risk neutral probability,  $\pi$ . In this section we relax this assumption, allowing for diverse opinions regarding the stochastic nature of K(I) and  $\pi$ .

In the case of heterogeneous expectations, the government is assumed to be less optimistic than the entrepreneur with regard to the stochastic nature of future cash flows for a given investment, K(I).

Accordingly, there are projects that are acceptable to the entrepreneur, on a pre-tax basis, given his expected value of K(I) and his risk-neutral probability  $\pi^E$ , but are rejected by the government, given its evaluation of the risk-neutral probability,  $\pi^G$ , and  $\pi^G < \pi^E$ . Within this group of projects, some will be rejected by the entrepreneur on an after-tax basis (if no subsidies are given) and some will remain acceptable. This case can be analyzed ex-ante from a social welfare prospective by comparing the opportunity costs associated with each party's investment decision.

Let us look at an investment opportunity from the perspective of the entrepreneurs (E) and the government (G) such that on a before tax basis NPV<sup>E</sup>( $\tau$ =0)> 0 and NPV<sup>G</sup>( $\tau$ =0)< 0. Let us denote by I\* and I\*\* the optimal investment levels before and after taxes, respectively. Note that I\* is the optimal investment if there are no corporate income taxes. However, if taxes are imposed (and no subsidies are provided), the entrepreneur will select to invest I\*\*, which is less than I\*. This illustrates the under-investment problem discussed above. Since NPV<sup>G</sup> < 0 the government rejects the project and does not provide subsidies.

We can describe the social welfare losses in the following table:

Who is Right?	Entrepreneur	Government
Decision of Entrepreneur		
Accept	$NPV^{E}(I^{*})-[NPV^{E}(I^{**})+G^{E}(I^{**})]$	$-[NPV^{G}(I^{**})+G^{G}(I^{**})]$
Reject	NPV <sup>E</sup> (I*)	0

Note: NPV(I\*) is on a before-tax basis while NPV(I\*\*) is on an after-tax basis in the Table.

With no subsidies, the entrepreneur can decide either to accept of reject the project. If the project is accepted, and if the entrepreneur has an ex-ante correct assessment of K(I), then a social

welfare loss of  $NPV^E(I^*)$  -  $[NPV^E(I^{**}) + G^E(I^{**})]$  is incurred due to under-investment.<sup>21</sup> (The superscripts E and G denote that the NPV is calculated according to parameters chosen by the entrepreneur and the government, respectively.  $G^E$  represents the present value of taxes).

However, if the project is accepted and  $I^{**}$  is invested but the entrepreneur is wrong and government is ex-ante right, the project yields a negative NPV, based on the government's assessment,  $NPV^G(I^{**}) < 0$ . However, since the government collects taxes with a present value of  $G^G(I^{**})$ , the social cost comprises the sum of the two components. This will be the social cost of making a wrong decision, which can be mitigated by the PV of taxes that may be collected  $G^G(I^{**})$ . If, due to lack of subsidies, the entrepreneur chooses to reject the investment, there is a social cost of  $NPV^E(I^*)$  if he is right, but zero if he is wrong.

If we follow the numerical example in Appendix A, the above case can be illustrated by assuming that the government applies a risk neutral probability of  $\pi^G$ =50%, and the entrepreneur a higher probability of  $\pi^E$ = 70%. The government's assessment leads it to reject the project. The entrepreneur estimating a higher probability, however, will accept the project even if taxes are paid and no subsidies apply. The entrepreneur rejects the project in the after-tax case if he believes that  $\pi$  = 60%. The social welfare losses for this numerical example are:

Who is Right?	Е	G
Decision of Entrepreneur		
Accept ( $\pi^E = 70\%$ )	$NPV^{E}$ (12.4) – $[NPV^{E}$ (116.9) + $G^{E}$ (116.8)]=	$-[NPV^G (1168) + G^G (1168)]$
	= 35.4 - [4.8 + 29.8] = 0.8	=-[-29.9+21.3]=8.6
Reject ( $\pi^E = 60\%$ )	$NPV^{E}(118.7) = 13.0$	0

<sup>&</sup>lt;sup>21</sup> In the brackets is the sum of total added value to the economy in present value terms, accrued by the entrepreneur after tax and subsidy basis  $NPV^E(I^{**})$ , and by the government  $G^E(I^{**})$ .

The question as to who is right on ex-ante basis is posited in terms of "which risk neutral probability is appropriate." As can be seen in the numerical example, the social cost incurred if the entrepreneur correctly assesses the risk and accepts the project is relatively small, though he may capture only a small fraction of the benefit (e.g., 4.8 while the government captures 29.8 for a total sum of 34.6, which is below the 35.4 value for the no-tax case). In our example, the Type I error (accepting an investment that should have been rejected) has a social cost of 8.7, while the Type II error (rejecting an investment that should have been rejected) has a social cost of 13.0.

It should be noted that social welfare in the above analysis is stated in terms of present value, taking into consideration the relevant risk neutral probabilities. It reflects the dilemma facing both the entrepreneur and the government in making investment decisions.

#### V. The Effect of Externalities

In Section III we analyze the case of subsidizing isolated projects, ignoring the possibility of economic externalities. By introducing the effect of potential spillovers on the profitability of investments in the private sector, it can be shown that a government may have an additional incentive to subsidize projects even if  $G^S<0$  in the single-firm case. Usually externalities or spillover effects are not explicitly modeled, since it is difficult to assess the impact of a firm's investment on the output of other firms. A number of empirical papers have tried to quantify the spillover effect of investment in R&D on local markets and internationally, see, for example Jaffe (1986), Bernstein and Nadiri (1989), Griliches (1992), Branstetter (1996), Nadiri and Kim (1996).

We incorporate externalities by extending our micro-economic analysis to a two firm model. It is assumed that firm A faces a cashflow function  $K_A(I_A)$ , which is a function of its investment  $I_A$ . In contrast, Firm B's cashflow function  $K_B=K_B(I_A,\ I_B)$ , is a function of its own investment  $I_B$ , as well as the investment of firm A,  $I_A$ . For example, A can be a high-tech firm developing new products and B, a provider of services to high-tech firms such as product design. Higher investment by A means higher expected cashflow for B.

To simplify exposition and analysis, without losing generality, it is assumed that  $I_A$  affects  $K_B$  but  $I_B$  does not affect  $K_A$ . If this is the case, the tax imposed on A may affect the productivity of B and taxes collected from firm B. In order to adequately analyze the economic decision facing the government, possible spillovers between the activities of A on B must be taken into account.

Given the above assumptions, firm A faces the same decision criterion as before (see equation (8)). Firm B faces the following problem

$$\max_{I_B} NPV_B(I_B \mid I_A),$$

where NPV is an extension of (2):

$$(11) \quad NPV_B(I_B \mid I_A) = I_B(1 - \alpha_B)(\tau_B \pi_B e^{-r} - 1) + \left(K_B(I_A, I_B) - \beta_B I_B\right)(1 - \tau_B)\pi_B e^{-r}$$

and  $\tau_B$  is the tax rate of B,  $\pi_B$  is the risk neutral probability of B,  $\alpha_B$  and  $\beta_B$  are the subsidy parameters for B. The spillover effect is embodied in the term  $K_B(I_A,I_B)$ .

In most cases the corporate tax rate is the same for all companies (i.e.  $\tau_A = \tau_B$ ). The trend worldwide is to deal with special situations on industry or regional basis, not directly through the tax code, but rather indirectly through grants or subsidies.

If due to taxes, assuming no subsidies, firm A decides to reject the investment  $I_A$ , it may have a negative impact on the performance of B, and the government may lose tax revenues from A, and from B. In addition, there may be a welfare loss due to firm A's decision to reject a socially desirable investment (i.e., an investment that has positive NPV without taxes) and an additional potential welfare loss due to an under-investment of firm B.

The present value of the government's claims from taxes net subsidies from firms A and B, if investments  $I_A$  and  $I_B$  are accepted, are:

(12) 
$$G_{A}^{s} = -\alpha_{A}I_{A} + \{(K_{A}(I_{A}) - \beta_{A}I_{A} - (1 - \alpha_{A})I_{A})\tau + \beta_{A}I_{A}\}\pi_{A}e^{-r}$$

$$G_{B}^{s} = -\alpha_{B}I_{B} + \{(K_{B}(I_{A}, I_{B}) - \beta_{B}I_{B} - (1 - \alpha_{B})I_{B})\tau + \beta_{B}I_{B}\}\pi_{B}e^{-r}$$

If firm B is not eligible for subsidies (i.e.  $\alpha_B = \beta_B = 0$ ) then the government's net claim on B is

$$G_B^s(I_A, I_B, 0) = \{(K_B(I_A, I_B) - I_B)\tau\}\pi_B e^{-r}$$

From the standpoint of social welfare, the government is justified to support project A if, project A is rejected solely on the basis of tax considerations, and if with subsidies the combined net receipts from A and B are positive,  $G_A^s + G_B^s > 0$ .

One can define the combined net present values of firms A and B from the new investments for the no-tax case as the "added value to the economy" resulting from the investments,  $H=NPV_A+NPV_B$ . The added value to the economy if taxes (and subsidies) are imposed can be defined as

(13) 
$$H^{S} = NPV_{A}^{S} + NPV_{B}^{S} + G_{A}^{S} + G_{B}^{S}$$

The added value H<sup>S</sup> measures the present value of the total net rent that accrue to the economy as a result of accepting the two investments.

Moreover, it can be shown that if firm A cannot charge firm B for its positive impact on the B's cash flow, and if this spillover effect is substantial, the government may be justified to encourage A to invest more than what otherwise would be optimal  $I_A$ \*. The government rather than Firm A can capture part of the spillover effect through taxes, and a socially welfare outcome can be achieved.

In order to illustrate these claims, let us continue the example from Section III and assume that firm A has a production function described in equation (6), with parameters specified in Appendix A. Firm B is assumed to have a production function similar to (6), but we add the assumption that the relevant investment is  $I_B$  and a fraction  $\delta$  of  $I_A$ . Accordingly,

(6B) 
$$K_{B}(I_{A}, I_{B}) = \frac{B_{1}}{1 + e^{e - f(I_{B} + \delta I_{A})}} - B_{2}$$

 $\delta$  represents the spillover factor, and  $B_1$ ,  $B_2$ , e and f are known parameters.

In Appendix B we provide a detailed numerical example that shows the optimal investments of A and B and their corresponding NPVs as well as G, the present value of the government's receipts for alternative tax ( $\tau$ =0, 10%, 20%, 30%) and subsidy rates ( $\alpha$ <sub>A</sub>=0, 10%, 20%, 30%, for  $\beta$ <sub>A</sub>=0,  $\alpha$ <sub>B</sub>= $\beta$ <sub>B</sub>=0). It is assumed that the spillover factor is  $\delta$ =0.1 and that firm B is less risky with a risk-neutral probability of  $\pi$ <sub>B</sub>=0.8.

It is shown, given the above assumptions, that without taxes and subsidies the combined NPV of firms A and B is 72.30. With a 30% tax rate, firm A rejects the investment and firm B does not benefit from A's aborted investment. In this case the NPV of B is only 23.23, while that of A is

zero, given the project's rejection and the government's receipts, in present value terms, come to 24.00. Hence, the added value to the economy (i. e.  $H^S = NPV_A^S + NPV_B^S + G_A^S + G_B^S$ ) is only 47.23.

With a subsidy rate  $\alpha_A$  of 10%, firm A invests 116.3 (compared to 118.7 for the no tax case), and adds 3.54 to its NPV. Firms A and B together attain an NPV<sup>S</sup> of 35.77, and G<sup>S</sup>=36.03.The total added value is 71.80. With a subsidy rate of 20% the total added value is 72.25 which approximates to the combined NPV for the no-tax case. Obviously, the allocation of values among A, B, and the government changes with each tax/subsidy combination.

Typically, the Government's economic objective is to collect a fixed amount  $G^S = \overline{G}$  (or,  $G_A^S + G_B^S = \overline{G}$ ), where  $\overline{G}$  is the assumed, fixed government budget and maintain an economically optimal level of investment,<sup>22</sup> i.e.  $I_A = I_A^*$ , and  $I_B = I_B^*$ . The assumption is that the optimal level of investment I\* yields optimal output from a social welfare perspective.<sup>23</sup>

Assume government must collect  $\overline{G}$  in order to finance its activities (and  $\overline{G}$  is determined exogenously to the model). The government can affect  $I_A$ ,  $I_B$  and  $G_A^S + G_B^S$  by changing both the tax rate,  $\tau$ , and the subsidy rate  $\alpha$ . The tax rate is assumed to be uniform for all firms and is more difficult to change than  $\alpha$ . In most cases, the subsidy rate can be altered with greater flexibility, and differentially across industries.

In Figure 2 we show the total added value (i.e.  $H^S = NPV_A^S + NPV_B^S + G_A^S + G_B^S$ ) and government receipts  $(G_A^S + G_B^S)$ , as a function of  $\alpha_A$ , the subsidy rate for firm A. At low levels of

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<sup>&</sup>lt;sup>22</sup> Where the asterisks denote the optimal level of investment for the no-tax case.

 $<sup>^{23}</sup>$  It should be noted that with a spillover effect that cannot be internalized, firm A invests less than the socially optimal level, since it ignores the effect on the NPV of firm B. In such a case subsidizing A may cause it to invest more than  $I_A^*$ , and thus capture, in social welfare terms, the impact of A on NPV<sub>B</sub>.

 $\alpha$ , firm A still rejects the investment. Only if  $\alpha$ >6.3% does firm A accept the investment, and as a result, both total added value,  $H^S$ , and government receipts (in present value terms),  $G^S$ , jump substantially, with the increase in  $H^S$  being higher than the increase in  $G^S$ .

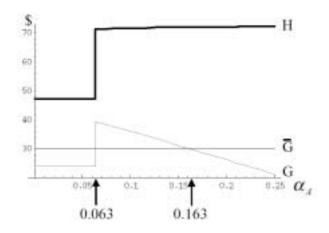


Figure 2. The total added value, H, and government value, G, as a function of the subsidy rate,  $\alpha_4$ .

In our numerical example, if the government has a fixed target of  $\overline{G} = 30$ , it can provide a subsidy rate of  $\alpha=16.3\%$ . In this case the resulting investment and values are as follows:

	Firm A	Firm B	Total
T	110.02	121.01	
Investment	118.92	121.01	
NPV <sup>S</sup>	9.70	32.44	42.14
G <sup>S</sup>	3.31	26.69	30.00
H <sup>S</sup>			72.14

Figure 3 depicts isocurves of  $G^S$  and  $H^S$  on the  $\alpha_A$ ,  $\alpha_B$  plane. The objective is to check the tradeoff between subsidizing A and B, and the optimal allocation of the subsidy budget in order

to maximize the social welfare function,  $H^S$ , subject to budgetary constraint,  $\overline{G}$ . The isocurve  $G^S$ =30 depicts all the pairs  $\alpha_A$ ,  $\alpha_B$  such that the total government claim is 30. In a similar way the isocurve  $H^S$ =72.05 depicts all pairs of  $\alpha_A$ ,  $\alpha_B$  (given optional investment decisions by equity holders) such that the total added value,  $H^S$ , is equal to 72.05. In Figure 3 we look for the optimal subsidy policy ( $\alpha_A$ ,  $\alpha_B$ ) for a given  $\overline{G}$ , that maximizes  $H^S$ . As can be seen, given our assumptions, the optimal policy is usually to support A and not B. This result emanates from the assumption that there is a spillover effect from A to B.

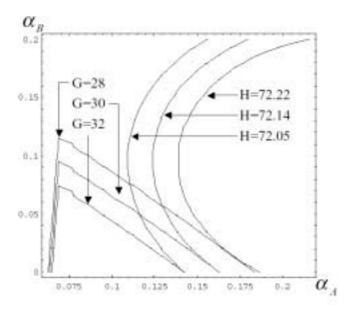


Figure 3. Isocurves of G and H on the  $\alpha_A$ ,  $\alpha_B$  plane.

For example, for  $\overline{G}=32$  the highest  $H^S$  that can be achieved is 72.05, and then  $\alpha_A$ =0.14 and  $\alpha_B$ =0. For  $\overline{G}=30$ , the highest total added value is  $H^S$ =72.14, in order to achieve it the government should subsidize A with  $\alpha_A$ =0.163 and the optimal subsidy to B is almost nil. For  $\overline{G}=28$ ,  $\alpha_A$ =0.163 and  $\alpha_B$ =0.015 and the highest total added value is  $H^S$ =72.2.

#### VI. Subsidizing High-Tech versus Low-Tech Projects

It is a common view that if a government decides to subsidize investments, it should subsidize high-tech companies rather then low-tech ones. The usual explanation is that high-tech companies face higher risk; in addition to the commercials risks encountered by all firms, they face a technological risk and the uncertainty of introducing new, unproven products into the market. Without subsidies, so the claim goes, companies will reject risky projects, opting for less risky alternatives instead. The rationale for subsidizing hi-tech ventures has also been supported by the claim that high-tech firms generate greater externalities than low-tech firms<sup>24</sup>.,

The differences between high-tech (HT) and low-tech (LT) firms can be introduced to the model of the firm in equation (1). The first distinction is reflected in the slope of the revenue function, K(I), which is presumed to be steeper for HT than for LT. For any given investment, I, it is assumed that HT yields greater revenues if successful. Secondly, the probability of HT to succeed is smaller than for LT. It should be noted that the certainty equivalent of the investment grant,  $\alpha$ , and royalty rate,  $\beta$ , is greater for a lower probability of success, since  $\alpha$  is received in advance as a function of the investment, while the obligation to pay,  $\beta$ , is contingent on future performance. The contingent royalty is ex-ante uncertain, and its present value is smaller the higher the uncertainty

The third difference is in the spillover effect, which for LT type firm is expected to be relatively small. The government is regarded as an economic agent, required to enhance welfare, under a given corporate tax code and with a limited, fixed budget to subsidize investment activity. In our partial equilibrium framework we focus on the decision of the government to

<sup>&</sup>lt;sup>24</sup> This viewpoint is strongly emphasized by the Department of Finance and Revenue, Canada in their report (December 1997).

subsidize either LT or HT, when the two firms are differentiated by the slope of K(I), the discount factor  $\pi$ , and the spillover factor,  $\delta$ .

In Figure 4 we describe the expected cashflow function, K(I), for HT and LT firms. For continuity we assume that LT is firm B from the previous section (see also Appendix B). Firm HT faces a production function as in (6), but it has steeper slope than LT ( $b_{HT} = 0.08$  and  $b_{LT} = 0.0368$ ). We also assume that the risk-neutral probability for LT is 0.8 compared to 0.35 for the HT firm. The spillover factor for HT,  $\delta$ , is assumed to be 0.1, i.e. each dollar invested in HT generates an additional 0.1 dollar at B. It is also assumed that no reverse spillovers from LT to HT exist.

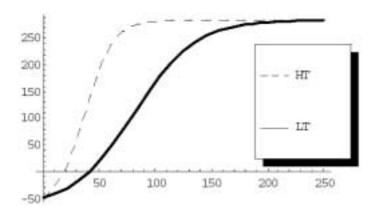


Figure 4. Comparison of profit functions K for HT and LT.

Table 1 summarizes the results for HT and LT under the above assumptions. With no taxes or subsidies, the two firms produce a risk adjusted NPV of 71.32 in total (see column 9). It should be noted that 71.32 is the socially optimal level if the two firms cannot collude. As can be seen in Figure 5, a higher level of NPV can be achieved if the spillover effect is internalized and

firm A invests more than 61.98. Inducing firm A to invest 63.70, for example, by providing it with a subsidy of  $\alpha$ =0.1, will increase the total NPV of the two firms from 71.32 to 77.78 by more than the subsidy paid to A by the government (6.37), so that H can be set at 71.40.

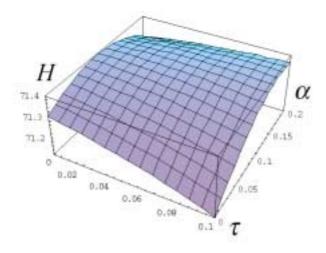


Figure 5. Total added values (H) for HT and LT companies as a function of the tax rate, and the subsidy rate.

If 30% taxes are imposed, HT will be indifferent to the investment opportunity, since its NPV is negligible (0.38). At  $\tau$  = 35% HT rejects the investment as the maximal NPV<sub>HT</sub> becomes negative, (-2.38). With  $\tau$  = 35%, total NPV is only 19.24 and the government collects 27.89 for a total net added value, H<sup>S</sup>, of 47.13. The government can restore the social welfare close to its maximum value by subsidizing HT investments by 10% as can be seen in the last row of Table 1.

**Table 1**: Optimal investment (I), NPV of the firm (NPV), government net benefit (G), and total added value (H) from the High-Tech (HT) and Low-Tech (LT) companies, for different tax rates (τ) and subsidies (α).

	HT			LT		Total			
	I (1)	NPV (2)	G (3)	I (4)	NPV (5)	G (6)	NPV (7)=(2)+(5)	G (8)=(3)+(6)	H (9)=(7)+(8)
$\tau=0, \alpha_{\rm HT}=0$	61.98	17.70	0	130.34	53.62	0	71.32	0	71.32
τ=0, α <sub>HT</sub> =10%	63.70	23.98	-6.37	130.16	53.80	0	77.78	-6.37	71.40
$\tau = 30\%, \alpha_{HT} = 0$	57.54	0.38	16.69	127.15	27.69	25.30	28.07	41.99	70.06
$\tau = 30\%$ , $\alpha_{HT} = 10\%$	59.46	5.65	11.83	126.96	27.84	25.35	33.49	37.20	70.86
$\tau = 35\%$ , $\alpha_{HT} = 0\%$	0	0(-2.38)	0	132.01	19.24	27.89	19.24	27.89	47.13
$\tau = 35\%$ , $\alpha_{HT} = 10\%$	58.46	2.71	14.60	126.16	23.55	29.43	26.25	44.03	70.28

Once again, it is better to subsidize HT than LT, given the assumptions outlined above and a budget constraint on G. This preference for high-tech is based on the higher risk associated with high-tech ventures, the greater sensitivity to tax rates, and on the outward direction of economic spillover. With a tax rate of 35% (and no subsidies) HT rejects projects, in which LT continues to invest. Hence, the probability of under-investment in high-tech industries is higher than in low-tech industries. Given the direction of economic spillovers, the economy risks losing more from the standpoint of cumulative social welfare if subsidies are withheld from high-tech enterprises.

#### VII. Summary and Conclusions

The paper focuses on the economic decisions of governments to subsidize investments in the private sector, and to discriminate among firms in its support programs. The presumption is that by taxing corporate profits, the government affects the investment decisions of firms and causes entrepreneurs to invest less than what is socially optimal. Therefore, investments desirable from a social standpoint may be rejected by shareholders. Withholding investment may also lead to the collection of fewer taxes.

Given corporate taxation, one possible economic solution to enhance public welfare and, simultaneously, increase government tax revenues is to subsidize investments. The primary advantage of investment subsidies over tax cuts stems from its flexibility and the capability it gives governments to discriminate among industries, firms and type of activity.

We show the conditions under which it is rational to subsidize investments initiated by the private sector. A scheme where the government provides investment grants and charges royalties from future revenues is analyzed. Such a scheme can reduce problems of asymmetric information between the owners of the firm and the government. It is shown that by calibrating the sizes of the grants  $(\alpha)$  and the royalty rates  $(\beta)$ , the government can partially restore socially optimal investment levels.

In the second part of the paper we look at the preference of supporting investments in high-tech, versus low-tech firms. Spillover effects (or externalities) are introduced to the model and analyzed. The basic presumption is that high-tech firms have more significant externalities than low-tech firms do and tend to suffer from problems of under-investment to a greater degree.

We show how the government should allocate subsidies between the two types of firms to enhance total social welfare, when it is constrained by the need to achieve fixed revenues. Under reasonable assumptions, the lion's share of support should be given to the high-tech firm. With the spillover effect, which cannot be captured by the originator, subsidies can augment the benefits of better investment decisions from the standpoint of social welfare.

Our analysis and conclusions are based on a novel approach to measure social welfare in an uncertain environment. We calculate the present value of cash flows to the entrepreneur plus the present value of net government receipts, discounted at rates that account for the investment risk. Consistent with the analysis of social welfare gains, we model externalities and measure their uncertain impact in terms of present value.

# Appendix A: A Numerical Example of the Effect of Taxes and Subsidies.

In this appendix we illustrate the investment decisions of firms under the different regimes: no tax case, corporate tax case and corporate tax plus subsidies case. Let the revenue function K(I) be

$$K(I) = \frac{350}{1 + e^{3 - 0.04I}} - 65.$$

This is an S-shaped revenue function depicted in Figure A1.

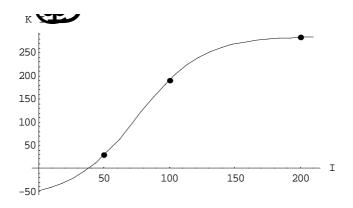


Figure A1.

For this function it does not pay to invest a small amount, and also it is sub-optimal to invest too much. The NPV of the project as a function of the investment I is shown by the thin line in Figure A2.

This NPV assumes no government support and no taxes. The optimal decision of the firm is to invest I=\$118.67 in the project and the maximal NPV achieved at this point is \$13.0 (assuming r=6% and  $\pi$ =60%).

A flat tax of 40% on all profits ( $\tau = 0.4$ ) is introduced. In this case the NPV function is shown by the dotted line on Figure A2. The firm will never undertake the project since its NPV is always negative.

The government can grant a subsidy, for example,  $\alpha$ =0.3 and which is partially repayable should the investment succeed,  $\beta$ =0.2. This means that 30% of the initial investment is given as a subsidized loan, with a conditional royalty rate of 20% returned in nominal terms from future cash flows. Should the project fail, the loan is forgiven. The NPV<sup>S</sup> function is given by the bold line in Figure A2.

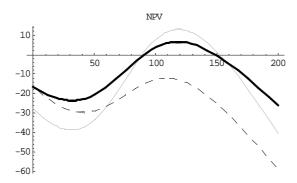


Figure A2.

The optimal decision of the firm in the tax and subsidy case is to invest I=\$118.1 in the project with an NPV of \$6.67. As can be seen by combining a subsidy with tax, the government can reach almost the same investment decision by the company as in the no tax case. The maximum of the bold line is lower than the one for the no-tax case. In our example the maximum NPV with subsidies falls short of optimality amounting to only \$6.67. The difference is the PV of net government revenues (taking into account subsidy payments, taxes and potential royalty revenues).

The table below shows the results of the investment optimization problem solved by a firm in a given tax regime.  $I_{optimal}$  denotes the optimal level of investment defined at the maximum of NPV<sub>firm</sub>; the government's claim in the net present value terms, G, is conditional on the NPV<sub>firm</sub>>0 (otherwise the firm will not take the project). We compare three basic cases: an economy without taxes and subsidies, an economy with corporate taxes alone, and an economy with both taxes and subsidies. The results are provided for different levels of probabilities of success,  $\pi$ . In the table it is assumed, as above, that the tax rate  $\tau$ =40%, and subsidies are  $\alpha$ =30% and royalties are  $\beta$ =20%.

	I <sub>optimal</sub>	NPV <sub>firm</sub>	G
		π	
		=	
		5	
		0	
		<b>%</b>	
No tax	111.8	-8.3	0
Tax	97.6	-27.2	16.3
Tax and subsidy	110.2	-7.8	-0.6
		π	
		=	
		6	
		0	
		<b>%</b>	
No tax	118.7	13.0	0
Tax	108.9	-12.0	23.6
Tax and subsidy	118.1	6.7	6.3

		π	
		=	
		7	
		0	
		%	
No tax	124.0	35.4	0
Tax	116.8	4.83	29.8
Tax and subsidy	124.3	21.9	13.5
		π	1
		=	
		8	
		0	
		%	
No tax	128.3	58.5	0
Tax	123.3	22.7	35.4
Tax and subsidy	129.4	37.7	20.7
		π	
		=	
		9	
		0	
		%	
No tax	132.0	82.0	0
Tax	129.0	41.3	40.6
Tax and subsidy	133.8	54.0	6.3
L			

From the table it is evident that investment subsidy is especially important for riskier projects, i.e. projects with lower probability of success. For example, with 40% tax rate the investment will be rejected for  $\pi$ =50% and  $\pi$ =60%, unless a subsidy is provided. However, for

 $\pi$ =70% and more, the investment is accepted even when a tax is collected and no subsidy is given to the firm.

# Appendix B: A Numerical Example of the Effects of Taxes, Subsidies and the Spillover Factor.

Let the revenue function for firm A,  $K_A(I_A)$ , be

$$K_A(I_A) = \frac{350}{1 + e^{3 - 0.04I_A}} - 65.$$

For this single firm case, assuming that  $\pi$ = 60% and r = 6%, we show in Appendix A that with no taxes and subsidies, the optimal investment is  $I_A$ \*= 118.7 and  $NPV_{\tau=0}$  = 13. With a flat tax rate of 40% the project is rejected by the shareholders since the NPV is negative.

Let us assume that firm B has a revenue function  $K_B(I_B,\,I_A)$  as follows:

$$K_B(I_B, I_A) = \frac{350}{1 + e^{3 - 0.0368(I_B + 0.1I_A)}} - 65$$
,

where the spillover factor  $\delta=0.1$ . We also assume that Firm B is less risky and assign a higher risk neutral probability  $\pi_B=0.80$ .

## The No-Tax Case

The optimal investment of A,  $I_A*=118.67$  has an effect on  $K_B$  equivalent to an additional investment of 11.87 by firm B. Given  $I_A*$  the optimal investment of B is  $I_B*=124.67$  with NPV<sub>B</sub>\*=59.29. The total NPV of A and B is therefore 72.30.

## The Tax Case

The following table shows the optimal investment of firm A and the resulting optimal investment of firm B (with a spillover factor of 10%), the corresponding NPVs and the government tax claim of taxes in present value terms for different levels of taxation.

Tax rate $\tau$	$I_A$ N	$PV_{A}(\tau, I_{A})$	$I_{B}(\tau,I_{A})$	$NPV_{B}\left( \tau ,I_{A},I_{B}\right)$	G
10%	116.97	6.59	123.87	50.14	15.35
20%	114.90	0.25	122.90	41.02	30.39
30%	112.28	-5.95	132.91	23.23	24.00
40%	108.86	-11.97	130.98	15.27	31.71

Note that in this table  $I_A(\tau)$  is the investment level at which NPV<sub>A</sub> is maximized. However, when NPV<sub>A</sub><0, the effective investment by A is 0, and therefore NPV<sub>B</sub> is a function of  $I_B$  alone. For example, at  $\tau$ =30% maximum NPV<sub>A</sub> is –5.95 and  $I_A$ =112.28; obviously firm A will reject the investment, and therefore its effective investment is zero. At a 20% tax rate firm A is (almost) indifferent to the investment, since NPV<sub>A</sub> is 0.25. By investing 114.9 firm A makes a contribution equivalent to a 11.49 investment of firm B. At a 30% tax rate, firm A rejects the investment and its effective investment is zero, making no contribution to firm B.

## The Tax and Subsidy Case

The table below is based on the above numerical example with the addition of a tax subsidy  $\alpha = 10\%$ , 20%, 30%. To simplify the example, and without loss of generality, we assume that the loan does not have to be repaid,  $\beta = 0$ .

In the tax case without subsidies, the investment is rejected when the tax rate is above 20% (at 20% NPV<sub>A</sub> is slightly above zero). At  $\tau=20\%$  the government collects taxes with present value of G=30.39 and the NPV of A and B collectively is NPV<sub>A</sub> + NPV<sub>B</sub> = 0.25+41.02=41.27. If a subsidy of  $\alpha=10\%$  is provided, firm A will restore its investment level to approximately the optimal, no-tax level of 118.75. In this case, the combined NPV of the firms comes to 10.61+41.35=51.96, and the government still collects 20.29 in present value terms.

If the government wants to maintain G at approximately 30 and is willing to provide a subsidy of 10%, then it should raise the tax rate to approximately 27%.

τ	α	$I_A$	NPV <sub>A</sub>	$I_{\mathrm{B}}$	NPV <sub>B</sub>	G
	10%	120.71	17.80	123.50	50.49	4.15
10%	20%	124.68	29.37	123.10	50.86	-7.83
	30%	128.98	41.34	122.67	51.25	-20.68
	10%	118.75	10.61	122.52	41.35	20.29
20%	20%	122.82	21.32	122.11	41.69	9.40
	30%	127.2	34.41	121.67	42.06	-2.36
	10%	116.30	3.54	121.28	32.23	36.03
30%	20%	120.50	13.37	120.86	32.56	26.32
	30%	125.00	23.56	120.40	32.91	15.73

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